Optimal Offering Strategies for Generation Companies Operating in Electricity Spot Markets

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***Abstract*—An unprecedented process of reforms has shaken the power industry during the last two decades. In order to sell the energy produced by their plants, many generation companies are now forced to prepare and submit daily offers to an electricity market under uncertainty about the offers submitted by their rivals. In this paper, we describe a methodology to prepare optimal offers for a generation company operating in a day-ahead market organized as a series of 24 hourly uniform-price multiunit double auctions. We explicitly consider the ability of the company to affect the price of electricity as well as the company's uncertainty about rivals' behavior.**

***Index Terms*—Electricity competition, market models, offering strategies, power generation scheduling.**

I. Introduction

THE power industry of an increasing number of countries is suffering an intense process of *reforms* oriented to the introduction of *competition* both at the wholesale and at the retail level [12]. Regulatory authorities in different regions have adopted a variety of approaches to introduce competition at the wholesale level. In general, energy can be traded through a number of market mechanisms with time scopes ranging from several years to a few hours prior to physical delivery. Although long- and medium-term markets are expected to gain importance, *electricity spot markets*, in which energy is traded for immediate delivery, have played the most relevant role and will continue to do so, given that they are considered as a *reference* for the rest of transactions.

In a significant number of cases the spot market is organized as a sequence of market mechanisms, typically including a *day-ahead* market, a *congestion management* procedure, an *adjustment* market and a market for *ancillary services*. Day-ahead markets, even if not mandatory, usually present large transaction volumes when compared to other spot market mechanisms. We focus on the problem that a generation company faces when preparing its offers for a day-ahead market, although we explicitly consider the flexibility provided by an adjustment market and the possibility of saving power for a reserves' market (Fig. 1).

We therefore assume that there are no significant transmission constraints, which simplifies the analysis and leads to results that can be generalized in a subsequent stage.

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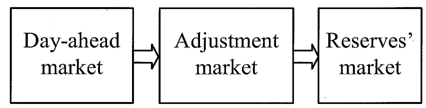

Fig. 1. Electricity spot market as a sequence of market mechanisms.

Spot market mechanisms are frequently organized as a series of hourly or semi-hourly *auctions* (e.g., the day-ahead market in the Spanish wholesale electricity market is organized as a set of twenty-four simultaneous hourly auctions). We assume that each spot market mechanism (i.e., the day-ahead market, the adjustment market and the reserves' market) is constituted by twenty-four hourly auctions. Each of these hourly auctions has the following characteristics:

- *Uniform priced*: All transactions are remunerated at the price given by the intersection of the aggregate offer curve and the aggregate demand curve.
- *Double*: Both offers from selling agents (e.g., generation companies) and bids from buying agents (e.g., distribution companies or retailers) can be submitted to each auction.
- *Multiunit*: Each seller can submit several sell offers and each buyer can submit several buy bids.
- *Sealed-bid*: Each agent is unaware of the offers or bids submitted by the rest of agents.

Each offer (bid) is defined by a quantity q and a price p, indicating the amount of energy q that the agent is willing to sell (purchase) at price p. Portfolio offering is permitted, so generation companies are not required to specify the particular unit corresponding to each offer. To derive the aggregate offer (demand) curve, offers (bids) are sorted by increasing (decreasing) prices and their quantities are accumulated.

We also assume that, after the clearing of each market mechanism, *information* about the submitted aggregate offer and demand curves is made publicly available.

In this paper, we present a decision-support tool for a generation company operating in an electricity spot market with the abovementioned characteristics. This tool provides optimal offers given a discrete probability distribution for the behavior of the rest of agents.

The paper is organized as follows. Section II provides a survey of recent modeling approaches adopted to represent competition in electricity markets and justifies our particular approach based on the idea of residual demand, which we fully describe in Section III. Section IV presents the manner in which we model the portfolio of the generation company, including not only its generation assets but also its position in long-term contracts and other strategic aspects. Section V explains how we solve the resulting large-scale two-stage stochastic program using Benders' decomposition. Section VI illustrates the performance of our approach with a numerical example. Finally, Section VII summarizes the main conclusions of our research.

II. Modeling Competition in Electricity Markets

The reform of the power industry has triggered the development of new conceptual models oriented to represent the interaction of agents in electricity markets [13]. Most of these models can be categorized into two main groups.

A. Models That Represent All the Generation Companies Participating in the Market of Study

Models in this first group can again be classified into two big families: *equilibrium* and *simulation* models.

Models based on the economic theory of equilibrium frequently represent generation companies as *Cournot* agents and are commonly formulated as *mixed complementarity problems* (or as systems of *variational inequalities*) thus benefiting from the existence of specific powerful commercial solvers [11], [18], [21]. Other equilibrium models assume that agents express their decisions in terms of offer curves and are based on the theory of *supply function equilibrium* (SFE) [15]. They are more sophisticated but significantly more difficult to solve [3], [9], [19]. Recently, models including *conjectural variations* have been proposed that are halfway between Cournot and SFE models [7], [8].

Simulation models are not based on the general theory of oligopoly but rather provide an ad hoc representation of the behavior of agents in a certain electricity marketplace, which limits the possibility of reaching general conclusions [17].

B. Models That Focus on a Particular Generation Company

Models in this second group can be categorized according to three aspects:

1) The manner in which they represent the spot market

   Two approaches can be adopted to represent the spot market auctions when only one company is considered:
   - If the company is unable to affect prices then each auction can be represented by its estimated clearing price [20].
   - In other case, the influence of the company on the clearing price must be explicitly considered [10]. This requires estimating the behavior of the rest of agents.

2) Their treatment of uncertainty

   A spot market model can be deterministic or probabilistic.

3) The detail with which they represent generation units

   Three different levels of detail can be adopted:
   - An aggregate model of the company's portfolio consisting of a unique cost curve and a maximum power output [22].
   - A model that distinguishes individual generation units but ignores intertemporal constraints such as ramp-rate limits or the evolution of hydro reserves [14].
   - A model that considers units' intertemporal constraints.

C. The Model Adopted in This Paper

In this paper, we only represent in detail the operation of the company of interest, including each of the company's generation units and other aspects of its portfolio, such as long-term contracts or specific long-term strategic decisions.

Our model of the spot market explicitly takes into account the influence that the company exerts on the price of electricity but also the uncertainty it faces. Uncertainty is an essential ingredient for the development of optimal offers [1].

III. Representing the Spot Market

A. A Representation Based on the Idea of Residual Demand

In each hourly auction, n, the amount of energy that a generation company is able to sell, q_n, depends on the clearing price, p_n. This is due to the combined effect of the demand at that price, $D_n(p_n)$, and the supply of the rest of generation companies at that price, $S_n^{\text{rest}}(p_n)$

$$q_n = D_n(p_n) - S_n^{\text{rest}}(p_n) = R_n(p_n) \tag{1}$$

where $R_n(\cdot)$ is the *residual demand* faced by the company in auction n. To obtain $R_n(p_n)$, the company only needs to know the demand, $D_n(p_n)$, and the aggregate offer, $S_n(p_n)$, as it can obtain $S_n^{rest}(p_n)$ by subtracting its own offer

$$S_n^{\text{rest}}(p_n) = S_n(p_n) - S_n^{\text{own}}(p_n). \tag{2}$$

Conversely, the clearing price can be expressed as a function of the company's sales

$$p_n = R_n^{-1}(q_n). \tag{3}$$

A function can also be derived for its *revenue*, ρ_n

$$\rho_n = p_n(q_n)q_n. \tag{4}$$

Given our hypotheses concerning information disclosure, the company can obtain its residual demand and its revenue function for each *past* hourly auction.

In order to represent these functions in our model, let us divide the range of quantities that the company can sell in auction n into J segments. These segments yield a piecewise linear representation for the inverse residual demand curve and for the revenue function, as shown in Fig. 2. It should be noticed that, in general, the revenue function is not convex.

Each segment j is defined by its lower bound, q_{jn}, and its upper bound, q_{j+1n}. We assign a binary variable u_{jn} to each segment j, such that $u_{jn} = 1$ if the company's sales in hour n are higher than q_{jn} and $u_{jn} = 0$ in other case. We also define a continuous variable v_{jn} to represent the portion of segment j that is filled. Segment j can only be used if segment $j-1$ is full.

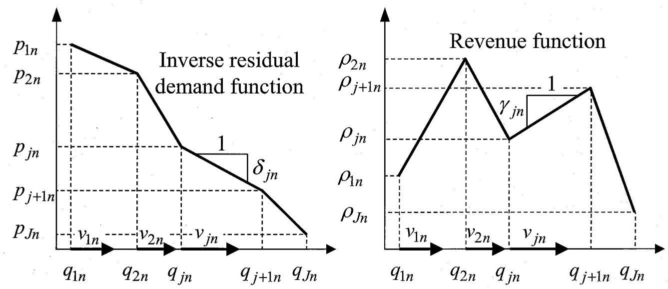

Fig. 2. Piecewise linear representation of residual demand and revenues.

Making use of the slopes of these piecewise linear functions, the following expressions provide the clearing price and the company's revenues for each level of sales q_n

$$p_n = p_{1n} + \sum_{j<J} \delta_{jn} v_{jn} \tag{5}$$

$$\rho_n = \rho_{1n} + \sum_{j<J} \gamma_{jn} v_{jn} \tag{6}$$

$$q_n = \sum_{j<J} v_{jn} \tag{7}$$

$$u_{j+1n}(q_{j+1n} - q_{jn}) \leq v_{jn} \leq u_{jn}(q_{j+1n} - q_{jn}), \ j<J \tag{8}$$

$$u_{jn} \leq u_{j-1n}, \ 1<j<J. \tag{9}$$

B. Representing Uncertainty in Each Market Mechanism

The generation company does not know its residual demand prior to submitting its offers to each auction. Therefore it must decide its offers based on historic information about the behavior of the rest of agents. We have assumed this historic information to be readily available.

If we focus on the twenty-four auctions that constitute the day-ahead market, the generation company must prepare its offers considering the *probability distribution* of the corresponding twenty-four residual demand curves. Although the problem of constructing this probability distribution exceeds the purpose of this paper, we suggest the following simplified approach. The company can look for recent days similar to the day of study by comparing the hourly demand of electricity expected for the day of study with that observed in recent days. Once a group of K similar days has been identified, the company can assume that the probability distribution for the market session of study is completely defined by these past K market sessions. This is equivalent to saying that the day-ahead market session has at most K equiprobable possible realizations (*finite support*).

Hence, we assume that in each of the spot market mechanisms, the generation company faces K possible series of twenty-four residual demand curves (Fig. 3).

Considering only one of the possible realizations, k, it is not difficult to determine the amount of energy that the company should sell in each of the twenty-four auctions so as to maximize its profit [2]. This would result in a vector of quantities, $(q_{1k}, \ldots, q_{nk}, \ldots, q_{24k})$ for each realization k.

Given the vector of quantities for each realization k, a vector of clearing prices results from the corresponding twenty-four residual demand curves, as shown in Fig. 4. Each pair (q_{nk}, p_{nk})

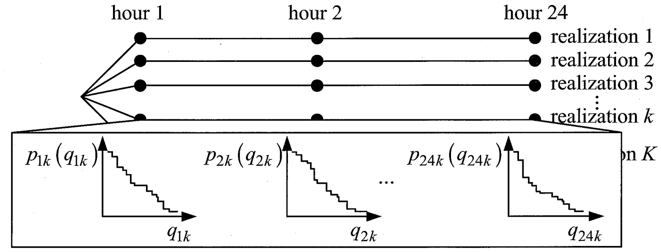

Fig. 3. Representing uncertainty in the spot market mechanisms.

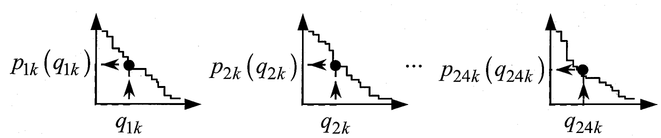

Fig. 4. Output decisions imply clearing prices for each realization k

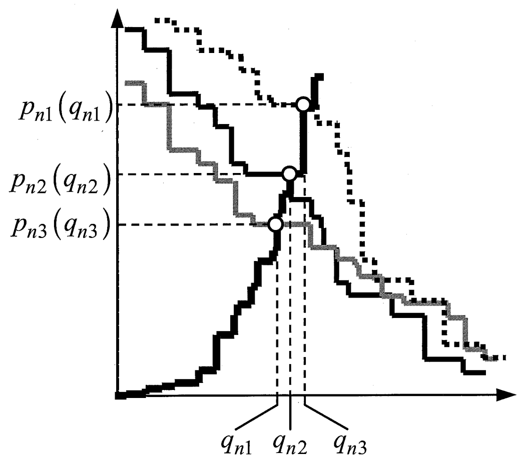

Fig. 5. Uncertainty in a particular hourly auction n.

can be considered as the offer decided by the company for auction n and realization k.

If we now focus on a particular auction, n, it is easy to understand that the K quantities and K prices decided by the company for that hour, although corresponding to different market outcomes, constitute the offer curve that the company must submit to that auction. Fig. 5 illustrates this idea for $K = 3$ and shows that the shape of the offer curve between each pair of contiguous residual demand curves is irrelevant. It also reveals that the decisions made by the company for the K different realizations of uncertainty are not independent, given that they must constitute nondecreasing offer curves.

To guarantee that the offer curves decided by the model are nondecreasing, the following condition must hold for each pair of offers (k, k') submitted for auction n:

$$(q_{nk} - q_{nk'})(p_{nk} - p_{nk'}) \geq 0, \ \forall n, \forall k, \forall k' > k. \tag{10}$$

These nonlinear constraints can also be formulated using linear expressions and binary variables

$$q_{nk} - q_{nk'} \geq -x_{nkk'} M^q, \ \forall n, \forall k, \forall k' > k \tag{11}$$

$$q_{nk'} - q_{nk} \geq -(1 - x_{nkk'}) M^q, \ \forall n, \forall k, \forall k' > k \tag{12}$$

$$p_{nk} - p_{nk'} \geq -x_{nkk'} M^p, \ \forall n, \forall k, \forall k' > k \tag{13}$$

$$p_{nk'} - p_{nk} \geq -(1 - x_{nkk'}) M^p, \ \forall n, \forall k, \forall k' > k \tag{14}$$

where M^q is a big quantity, M^p is a big price and $x_{nkk'}$ is a binary variable. If $x_{nkk'} = 1$ then $q_{nk} \geq q_{nk'}$ and $p_{nk} \geq p_{nk'}$. Conversely, if $x_{nkk'} = 0$ then $q_{nk} \leq q_{nk'}$ and $p_{nk} \leq p_{nk'}$. These constraints establish a link between the offers decided for the different realizations and complicate the search for an optimal strategy. They can be summarized in the following compact formulation, expressing that the quantities offered in each hour n must belong to the set of feasible offer curves, Q

$$\{q_{nk}, \forall k\} \in Q, \quad \forall n. \tag{15}$$

In this context, calculating the company's expected revenues or the expected clearing price for a certain auction n is straightforward

$$\mathrm{E}[\rho] = \sum_k \pi_k \sum_n \rho_{nk}(q_{nk}) \tag{16}$$

$$\mathrm{E}[p_n] = \sum_k \pi_k p_{nk}(q_{nk}) \tag{17}$$

where π_k is the probability of the k-th market realization.

C. Representing the Sequence of Spot Market Mechanisms

According to our assumptions, the company must make its decisions in several stages. In the first stage, the company decides its offers for the day-ahead market. After the clearing of the day-ahead market, in the second stage, the company can correct its schedule with its offers for the adjustment market. Subsequently, the company determines the reserve that its units can provide and offers it to the ancillary services' market. A final generation schedule results from this sequence of market mechanisms.

In practice, the volumes traded in the spot market mechanisms diminish as the moment of physical delivery gets nearer. For example, in the Spanish spot market, the volume of energy traded in the adjustment market is usually between a 10 and a 20% of the volume traded in the day-ahead market. Similarly, the reserve market is less relevant than the adjustment market. This suggests simplifying the representation of the adjustment market and the reserves' market when deciding the offers for the day-ahead market. Specifically, we assume that each realization of the day-ahead market is accompanied by a single possible realization of the adjustment market and the reserves' market (Fig. 6).

We therefore represent the participation of a generation company in the spot market from the point of view of the day-ahead market as a *two-stage stochastic decision process*. Uncertainty is only present in the first stage, where the company decides its offers for the day-ahead market. Given the possible outcomes of the day-ahead market, in the second stage the company evaluates its possible sales in the adjustment market and the reserves' market and derives a final schedule for its generating units. It must be noticed that, while the decisions taken for the day-ahead market take the form of offer curves that can be directly submitted to the market operator, the decisions for the adjustment market and the reserves' market as well as the generation schedule are only preliminary and will have to be revised in further detail once the day-ahead market has been cleared. Consequently, the revenue functions used to estimate the company's revenues in the adjustment market and in the reserves' market can be modeled in a simplified manner. A convenient approach is to estimate them using piecewise linear convex functions.

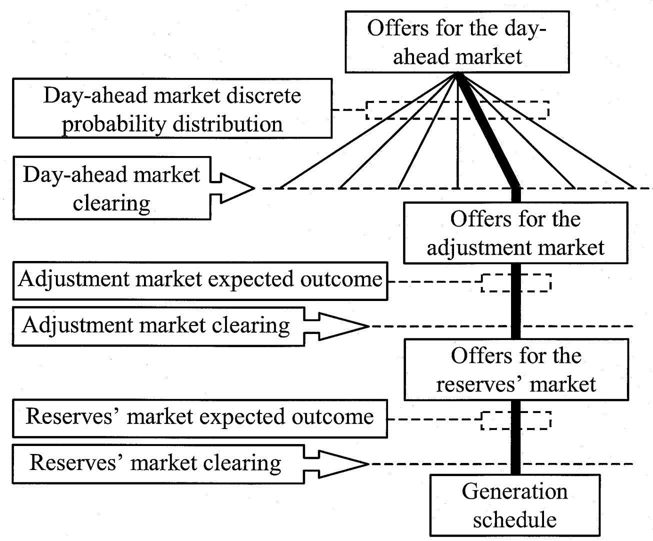

Fig. 6. Decision process in the spot market assumed in this paper.

IV. Representing the Company's Portfolio

A. Generating Units

In this paper, we represent the operation of the company's generating units avoiding the use of nonconvex expressions (e.g. we do not make commitment decisions). Equations are formulated for each scenario, k, to obtain a feasible schedule that supports the company's sales in the spot market.

We approximate the production costs of thermal unit t in each hour n and each market situation k as a linear function

$$c_{nk}^t = o^t q_{nk}^t + f^t \left(\beta^t u_{nk}^t + \alpha^t \frac{q_{nk}^t}{k^t} \right), \quad \forall t, \forall n, \forall k \tag{18}$$

where o^t are unit t's variable O&M costs, in Euro/MWh, q_{nk}^t is the net output of unit t, in megawatts, f^t is the fuel cost, in Euro/Tcal, β^t is the independent term of the heat rate function, in Tcal/h, $u_{nk}^t \in \{0, 1\}$ is the commitment state (on/off) for unit t in hour n and situation k, α^t is the linear term of the heat rate function, in Tcal/MWh, and k^t is the self-consumption coefficient of unit t, in p.u. We assume that commitment decisions have already been made with a one-week time scope and that u_{nk}^t enters as input data [2]. Thermal units also have a gross maximum capacity $\overline{q}^t$, in MW, a gross minimum stable output $\underline{q}^t$, in megawatts, and a ramp-rate limit, l_t, in megawatt hours

$$\underline{q}^t k^t u_{nk}^t \leq q_{nk}^t + r_{nk}^t \leq \overline{q}^t k^t u_{nk}^t, \quad \forall t, \forall n, \forall k \tag{19}$$

$$-l^t \leq q_{nk}^t - q_{n-1k}^t \leq l^t, \quad \forall t, \forall n, \forall k \tag{20}$$

where r_{nk}^t is the amount of reserve provided by unit t, in MW.

All these constraints can be summarized by expressing that the schedule decided for unit t under market scenario k must belong to its set of feasible schedules, Q^t:

$$\{q_{nk}^t, r_{nk}^t, \forall n\} \in Q^t, \quad \forall t, \forall k. \tag{21}$$

Our model manages hydro reserves in an aggregate manner by integrating hydro plants located in the same river basin into an equivalent hydro unit, h. The detail of the hydro network can be considered in a subsequent decision stage in order to derive a more precise hydro schedule. We consider a constant energy/flow ratio for each equivalent hydro unit and express hydro reserves in terms of stored energy, in MWh. Equivalent units can also operate in pumping mode. The state of each equivalent reservoir h is evaluated as follows:

$$w_{nk}^h = w_{n-1k}^h - \frac{q_{nk}^h}{k^h} + i_n^h - s_{nk}^h + \eta^h b_{nk}^h, \quad \forall h, \forall n, \forall k \quad (22)$$

where w_{nk}^h is the energy stored by unit h at the end of hour n in market situation k, in MWh, q_{nk}^h is its net output in hour n and situation k, in MW, k^h is its self consumption coefficient, in p.u., i_n^h are the net inflows it receives in hour n, in MWh, s_{nk}^h is the energy spilt in hour n and situation k, in MWh, b_{nk}^h is the energy pumped in hour n and situation k, in MWh and η^h is the performance of the pump-turbine cycle, in p.u.

Each unit has gross maximum generation and pumping capacities, $\overline{q}^h$, $\overline{b}^h$, both in MW. Its reservoir has a maximum and a minimum operating level, $\overline{w}^h$, $\underline{w}^h$, in MWh

$$0 \le q_{nk}^h + r_{nk}^h \le k^h \overline{q}^h, \quad \forall h, \forall n, \forall k \quad (23)$$

$$0 \le b_{nk}^h \le \overline{b}^h, \quad \forall h, \forall n, \forall k \quad (24)$$

$$0 \le s_{nk}^h, \quad \forall h, \forall n, \forall k \quad (25)$$

$$\underline{w}^h \le w_{nk}^h \le \overline{w}^h, \quad \forall h, \forall n, \forall k \quad (26)$$

where r_{nk}^h is the amount of reserve provided by unit h, in MW. In this paper, we assume that unit h has a certain amount of energy, w_0^h, available for the planning horizon. A medium-term hydrothermal model can determine this energy.

All these constraints can be summarized by expressing that the schedule decided for unit h under market scenario k must belong to its set of feasible schedules, Q^h:

$$\left\{q_{nk}^h, b_{nk}^h, r_{nk}^h, s_{nk}^h, w_{nk}^h, \forall n\right\} \in Q^h, \quad \forall h, \forall k. \quad (27)$$

B. Forward Contracts

In many cases, a generation company has the possibility of selling part of its production through long-term contracts. These contracts, in their most basic form, consist of an agreement to sell a certain amount of energy for a fixed price at certain hours and at a certain node of the network. Before entering into one of these contracts, the company must certainly evaluate the profit it expects to obtain from it. In the short term, however, the company must evaluate the influence that its portfolio of long-term contracts exerts on the profit it expects to obtain in the spot market. From this perspective it is important to consider whether the settlement of these contracts is physical or financial.

A long-term contract that has to be physically settled implies the obligation to supply a certain amount of energy during a number of hours. The company must take this into account when scheduling its generating units. These are commonly known as *physical bilateral contracts* (PBC's). Let $\mathrm{C^P}$ be the set of PBC's signed by the generation company and let c be one of them. The amount of energy that the company has agreed to serve in hour n as a result of this contract is q_n^c MWh and the price that the company will be paid is p_n^c Euro/MWh. With this contract, the company's revenue increases in $p_n^c q_n^c$ Euro. This payment cannot be modified by the company in the spot market. In contrast, in a financial contract, one party pays to the other the difference between a fixed price and the spot price for a certain hour n. These are commonly known as *contracts for differences* (CfDs). Let $\mathrm{C^D}$ be the set of CfDs signed by the generation company. A CfD, c, for a quantity q_n^c and a price p_n^c does not affect the company's generation schedule. However, its net revenue, $(p_n^c - p_{nk})q_n^c$, depends on the spot price expected for situation k, p_{nk}, and on the company's strategy in the spot market.

C. Energy and Reserve Balance Equations

As has been mentioned, in each scenario k the company decides to sell an amount of energy q_{nk}^d in the n-th hourly auction of day-ahead market and a net amount q_{nk}^a in the n-th hourly auction of the adjustment market. The company has also sold an amount q_n^c through each physical bilateral contract $c \in \mathrm{C^P}$. In order to guarantee that the company is able to produce this energy with its generating units, the following *energy balance equation* must be formulated:

$$q_{nk}^d + q_{nk}^a + \sum_{c \in \mathrm{C^P}} q_n^c = \sum_t q_{nk}^t + \sum_h q_{nk}^h - b_{nk}^h, \ \forall n, \forall k. \quad (28)$$

The company has also sold an amount of reserve q_{nk}^r in each hourly auction of the reserve market and has the obligation of providing this reserve. A *reserve balance equation* must then be formulated:

$$q_{nk}^r = \sum_{t \in \mathrm{T}} r_{nk}^t + \sum_{h \in \mathrm{H}} r_{nk}^h, \quad \forall n, \forall k. \quad (29)$$

D. Long-Term Guidelines

Traditional short-term planning tools such as *unit-commitment* or *economic-dispatch* models include constraints that orient their results toward medium-term objectives. For example, a volume of available hydro resources (or alternatively, a water-value curve) is typically specified according to the results of a medium-term *hydrothermal-coordination* model. This prevents short-term models from suggesting the naïve decision of using all hydro resources.

Similarly, short-term models used to decide the amount of energy that a company must sell in a certain auction, typically suggest reducing the company's output in order to increase the clearing price well above its marginal costs. This effect can be justified with a simplified expression of the company's profit:

$$P = p(q)q - c(q) \quad (30)$$

where $c(q)$ is the cost of the energy sold by the company, q, and $p(q)$ is the clearing price. The profit-maximizing output must fulfill the following first-order optimality condition:

$$\frac{\partial P}{\partial q} = p(q) + \frac{\partial p(q)}{\partial q} q - c'(q) = 0. \quad (31)$$

Hence, the optimal difference between the clearing price, $p(q)$, and the company's marginal costs, $c'(q)$ increases with the absolute value of the slope of the residual demand

$$p(q) - c'(q) = -\frac{\partial p(q)}{\partial q} q = \left|\frac{\partial p(q)}{\partial q}\right| q. \quad (32)$$

This difference is usually known as *price markup*.

In an auction in which the residual demand is very steep and the company's output is expected to be high (a typical situation in on-peak hours) the model would blindly tend to reduce the company's production so as to increase the difference between the clearing price and the company's marginal costs. However, this allows its rivals to sell more quantity at a higher price. If the company repeatedly gives up its position during on-peak hours, its rivals will tend to increase their market shares, so that in the long run prices will return to their original level and the company will have lost its market position. The possibility of suffering market-power mitigation measures from the regulator is another adverse consequence of this myopic behavior. A simple approach to avoid these undesirable effects is to take into consideration the future value of the company's current market position by adding a new term to the expression of the company's profit

$$P = p(q)q - c(q) + \sigma \frac{q}{q^T} \tag{33}$$

where σ is the value of the company's market share, expressed in Euro per unit, and q^T is the total expected trading volume, in MWh. σ can be obtained from a medium-term strategic model including a minimum market share constraint.

E. Expected Profit

The final expression of the company's expected profit including all the abovementioned contributions is

$$\mathrm{E}[P] = \sum_k \pi_k \Bigg[\sum_n \Big\{ \rho_{nk}^d \left(q_{nk}^d\right) + \rho_{nk}^a \left(q_{nk}^a\right) + \rho_{nk}^r \left(q_{nk}^r\right) + \sum_{c \in \mathrm{C^P}} p_n^c q_n^c + \sum_{c \in \mathrm{C^D}} \left(p_n^c - p_{nk}^d\right) q_n^c + \sigma_n^d \frac{q_{nk}^d}{q_n^{Td}} - \sum_t c_{nk}^t \left(q_{nk}^t\right) \Big\} \Bigg] \tag{34}$$

where ρ_{nk}^d, ρ_{nk}^a and ρ_{nk}^r are the revenues obtained by the company in the day-ahead market, in the adjustment market and in the reserves' market, respectively, in hour n and scenario k. We have only considered the value of the company's share in the day-ahead market.

V. Solution Strategy: Benders Decomposition

The following is a compact formulation for the problem described in previous sections:

$$\begin{aligned}
&\underset{\substack{q_{nk}^d, q_{nk}^a, q_{nk}^r, \\ q_{nk}^t, r_{nk}^t, q_{nk}^h, b_{nk}^h, r_{nk}^h}}{\mathrm{Max}} \quad \mathrm{E}[P] \\
&\text{s.t.}: \left\{q_{nk}^d, \forall k\right\} \in Q^d, \quad \forall n, \\
&\quad \left\{q_{nk}^t, r_{nk}^t, \forall n\right\} \in Q^t, \quad \forall k, \forall t, \\
&\quad \left\{q_{nk}^h, b_{nk}^h, r_{nk}^h, s_{nk}^h, w_{nk}^h, \forall n\right\} \in Q^h, \quad \forall k, \forall h, \\
&\quad q_{nk}^d + q_{nk}^a + \sum_{c \in \mathrm{C^P}} q_n^c = \sum_t q_{nk}^t + \sum_h q_{nk}^h - b_{nk}^h, \quad \forall n, \forall k, \\
&\quad q_{nk}^r = \sum_{t \in \mathrm{T}} r_{nk}^t + \sum_{h \in \mathrm{H}} r_{nk}^t, \quad \forall n, \forall k.
\end{aligned}$$

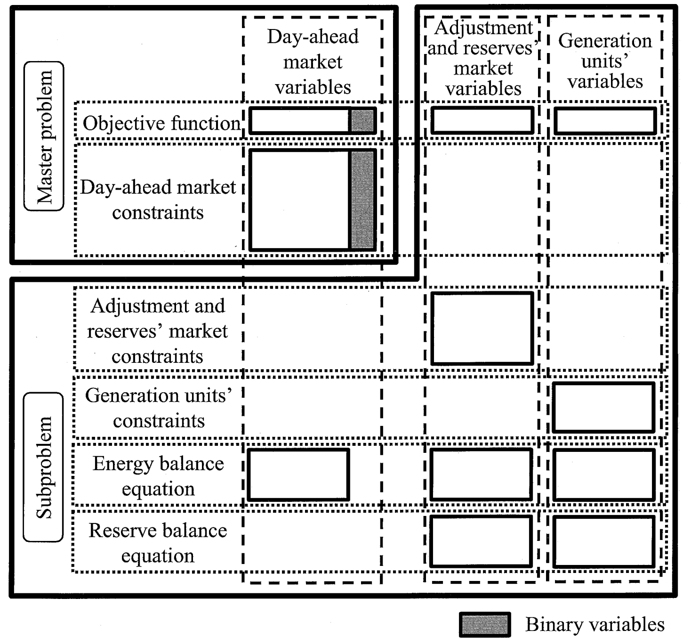

Fig. 7. Benders decomposition applied to our problem.

We have formulated this problem using linear expressions except for the *binary variables* used to evaluate the company's revenue in the day-ahead market and to guarantee that the decisions made for the day-ahead market constitute nondecreasing offer curves. Hence, complicating variables only appear in the day-ahead market.

Benders decomposition is particularly amenable to this problem structure. It decomposes the problem into a master problem (MP) that can take any form (including binary variables) and a subproblem (SP) that must be convex [4]. When dealing with two-stage stochastic programs [5], a natural approach is to include first-stage decisions in the MP and second-stage decisions in the SP, as shown in Fig. 7.

Hence, problem MP is formulated as follows:

$$MP \begin{cases}
\underset{q_{nk}^d, \theta_k}{\mathrm{Max}} \sum_k \pi_k \Bigg[\sum_n \Big\{ \rho_{nk}^d \left(q_{nk}^d\right) + \sum_{c \in \mathrm{C^D}} \left(p_n^c - p_{nk}^d \left(q_{nk}^d\right)\right) q_n^c + \sum_{c \in \mathrm{C^P}} p_n^c q_n^c + \sigma_n^d \frac{q_{nk}^d}{q_n^{Td}} \Big\} + \theta_k \Bigg] \\
\text{s.t.}: \left\{q_{nk}^d, \forall k\right\} \in Q^d, \quad \forall n, \\
\qquad \theta_k \le {\theta_k}^{\nu} + \sum_n {\lambda_{nk}}^{\nu} \left(q_{nk}^d - {q_{nk}^d}^{\nu}\right), \quad \forall k, \nu \in \mathrm{V^O}
\end{cases}$$

where θ_k is an *approximation* of the maximum profit that the company obtains in the second-stage if situation k occurs. This approximation is constructed as follows.

Given the level of sales ${q_{nk}^d}^{\nu}$ decided in iteration ν by the master problem for each auction n of the day-ahead market, the following linear program, SP_k, provides the second-stage maximum profit if market situation k occurs:

$$\mathrm{SP}_k \begin{cases} \theta_k^v = \underset{\substack{q_{nk}^a, q_{nk}^r, \\ q_{nk}^t, r_{nk}^t, q_{nk}^h, b_{nk}^h, r_{nk}^h}}{\mathrm{Max}} \quad \rho_{nk}^a\left(q_{nk}^a\right) + \rho_{nk}^r\left(q_{nk}^r\right) \\ \quad - \sum_t c_{nk}^t\left(q_{nk}^t\right) \\ \text{s.t.}: \left\{q_{nk}^d\right\} \in Q^d, \quad \forall n, \\ \quad \left\{q_{nk}^t, r_{nk}^t, \forall n\right\} \in Q^t, \quad \forall t, \\ \quad \left\{q_{nk}^h, b_{nk}^h, r_{nk}^h, s_{nk}^h, w_{nk}^h, \forall n\right\} \in Q^h, \quad \forall h, \\ \quad -q_{nk}^a + \sum_{c \in \mathrm{C^P}} q_n^c + \sum_t q_{nk}^t + \\ \qquad \sum_h q_{nk}^h - b_{nk}^h = q_{nk}^d, \quad \forall n, \\ \quad q_{nk}^r = \sum_t r_{nk}^t + \sum_h r_{nk}^h, \quad \forall n. \end{cases}$$

The dual variable of the energy balance equation, ${\lambda_{nk}}^{\nu}$, indicates how the company's maximum second-stage profit deviates from ${\theta_k}^{\nu}$ when slight changes are introduced in the first-stage decisions, $\Delta {q_{nk}^d}^{\nu} = q_{nk}^d - {q_{nk}^d}^{\nu}$. This is the approximation provided by Benders' cuts

$$\theta_k \le {\theta_k}^{\nu} + \sum_n {\lambda_{nk}}^{\nu}\left(q_{nk}^d - {q_{nk}^d}^{\nu}\right), \quad \forall k, \nu \in \mathrm{V^O} \tag{35}$$

where $\mathrm{V^O}$ is the set of cuts currently available. It is evident that this is an *outer linearization* of the second-stage profit function (also known as *recourse function*.) Therefore, it suggests higher profits than those that are obtained when SP_k is solved. However, after a certain number of iterations, the number of cuts is large enough to provide an exact approximation of the recourse function for situation k in the region of interest. When this happens, the solution provided by MP coincides with that of the original problem.

Benders algorithm is then summarized as follows:

Step 1: Set $\nu = 0$.
Step 2: If $\nu = 0$ then set $\theta_k = 0$.
  Set $\nu = \nu + 1$.
  Solve MP: ${q_{nk}^d}^{\nu}$ and θ_k are obtained.
Step 3: Loop in k:
   Solve SP_k: ${\theta_k}^{\nu}$ and ${\lambda_{nk}}^{\nu}$ are obtained.
   Add a new cut to MP.
  End loop.
Step 4: Check for convergence. If ${\theta_k}^{\nu} < \theta_k$, go to step 2.

It must be noticed that in each iteration K cuts are generated. This is the *multicut* version of Benders decomposition applied to a two-stage stochastic program.

An interesting feature of problem MP is that the offers chosen for different hourly auctions of the day-ahead market are only linked by the cuts approximating the recourse function for each scenario, θ_k. This function can also be expressed as the sum of hourly recourse functions, as follows:

$$\theta_k = \sum_n \theta_{nk} \le \sum_n \left\{ {\theta_{nk}}^{\nu} + \sum_k {\lambda_{nk}}^{\nu}\left(q_{nk}^d - {q_{nk}^d}^{\nu}\right)\right\} \tag{36}$$

where ${\theta_{nk}}^v$ is the value of the recourse function for hour n given by subproblem SP_k at iteration v.

It is because of the recourse function that problem MP is not separable into hourly problems of the form

$$\mathrm{MP}_n \begin{cases} \underset{q_{nk}^d, \theta_k}{\mathrm{Max}} \sum_k \pi_k \Big[\rho_{nk}^d\left(q_{nk}^d\right) + \sum_{c \in \mathrm{C^D}} \left(p_n^c - p_{nk}^d\left(q_{nk}^d\right)\right) q_n^c \\ \qquad + \sigma_n^d \frac{q_{nk}^d}{q_n^{Td}} + \theta_{nk}\Big] \\ \text{s.t.}: \left\{q_{nk}^d, \forall k\right\} \in Q^d, \\ \quad \theta_{nk} \le {\theta_{nk}}^{\nu} + {\lambda_{nk}}^{\nu}\left(q_{nk}^d - {q_{nk}^d}^{\nu}\right), \quad \forall k, \nu \in \mathrm{V^O}. \end{cases}$$

However, it is easy to see that any feasible solution for MP_n, $\forall n$, is also feasible for MP. Therefore, an initial feasible integer solution for problem MP can be obtained in each iteration of Benders algorithm by first solving MP_n, $\forall n$.

TABLE I
GENERATION UNITS OWNED BY THE COMPANY OF STUDY

Type	Plants	Gross maximum output (MW)	Minimum stable output (MW)	Maximum pumping capacity (MW)	Hydro resources (MWh)
Nuclear	3	2940	2940		
Coal	20	6169	2999		
Oil/gas	15	1440	366		
Hydro	16	4957		1614	12250

VI. NUMERICAL EXAMPLE

The model formulated in this paper has been implemented in GAMS, together with Benders algorithm [6].

We have solved the case of a large fictitious generation company facing eleven scenarios in the Spanish electricity spot market session that took place on October 24th, 2001. This company owns the generation units indicated in Table I.

The eleven possible realizations for the day-ahead market have been obtained searching for similar spot market sessions among the previous 23 days of October 2001. This search has been carried out by grouping the 24 days into four clusters according to their demand profile, as shown in Table II:

We have obtained residual-demand data from the Spanish Market Operator [16]. Each curve is approximated by a 20-point piecewise linear function ranging from 0 to 150 Euro/MWh. Fig. 8 shows the eleven possible residual demand realizations for the 5th and the 12th day-ahead market auctions.

We have obtained adjustment market data from the first of the six sessions of the Spanish intraday market. We have not considered the reserve market in this study case.

CPLEX 7.5 is unable to find a feasible integer solution for the resulting problem whose size, together with those of problem MP and problems SP_k, is shown in Table III:

TABLE II
CLUSTERING ANALYSIS TO SEARCH FOR SIMILAR HISTORIC DAYS

October 2001																								
Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Day	M	T	W	Th	F	Sa	S	M	T	W	Th	F	Sa	S	M	T	W	Th	F	Sa	S	M	T	W
Cluster	2	4	4	4	4	3	1	2	4	4	4	1	3	1	2	4	2	4	4	3	1	2	4	4

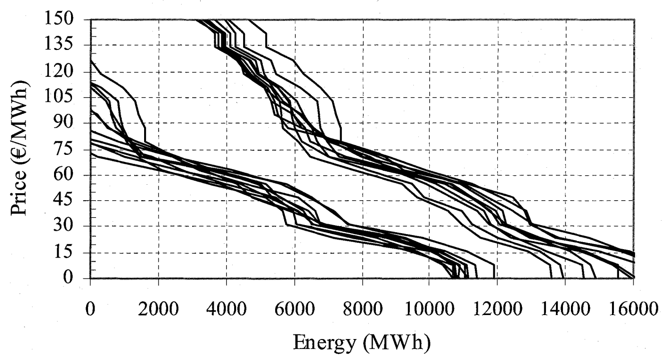

Fig. 8. Residual demand realizations for two hourly auctions.

TABLE III
PROBLEM SIZES IN THE FIRST ITERATION

Problem	Constraints	Variables	Binary variables
P	99758	91043	10529
MP	30325	15813	10529
SP_k (all)	69432	75240	

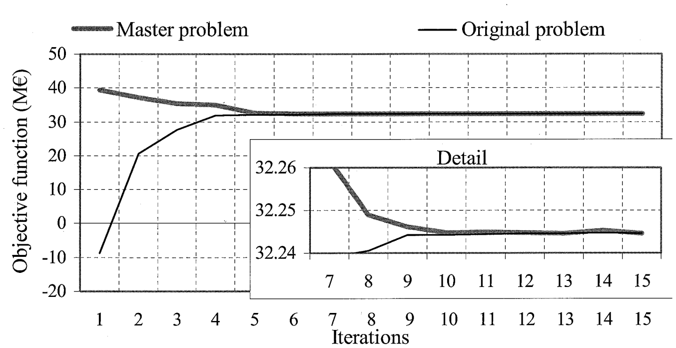

Fig. 9. Evolution of the objective function in Benders' algorithm.

Problem MP is still hard to solve. Hence, in each iteration we first solve problems MP_n to obtain an initial feasible point.

15 iterations of Benders' algorithm and 93 min in a PC 1.2 GHz 512 MB were required to solve this problem. Fig. 9 depicts the evolution of the objective function of problem MP, which provides an upper bound for the original problem.

The solution yields the optimal offers that the company must submit to each of the day-ahead market auctions. Fig. 10 shows the offers corresponding to the 5th and the 12th auctions.

These results can be analyzed from a different perspective. Fig. 11 represents the hourly quantities that the company would sell in each of the day-ahead market realizations:

Similarly, Fig. 12 represents the clearing prices that would result in each of the day-ahead market outcomes.

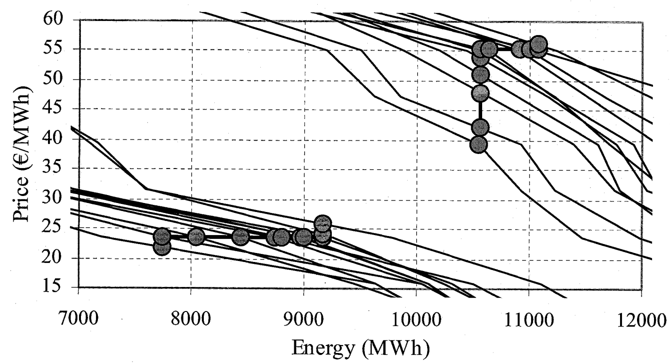

Fig. 10. Offers for two of the day-ahead market auctions.

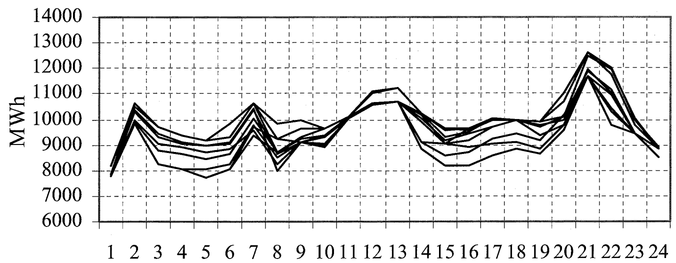

Fig. 11. Hourly quantities sold by the company in each scenario.

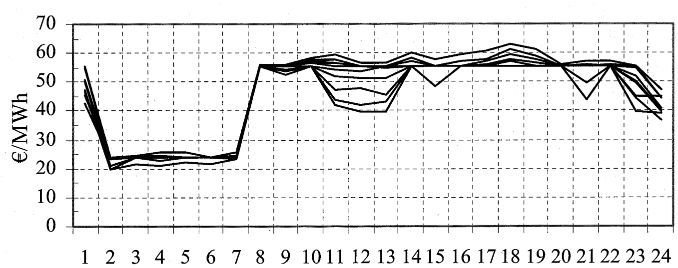

Fig. 12. Clearing prices for each day-ahead market scenario.

An interesting link can be established between Figs. 10, 11, and 12. Fig. 10 shows that the offer curve obtained for the 5th hourly auction is quite flat, thus making the company rather uncertain about the amount of energy that it will finally sell. This is confirmed by Fig. 11, where the company's eleven possible levels of sales for the 5th hour are very different. In contrast, the range of possible clearing prices for the 5th auction is rather narrow (Fig. 12). A similar analysis can be performed for the 12th auction, for which the company's offer curve is rather steep. This highlights the importance of the shape of offer curves and suggests using risk measures to limit the lowest revenue that the company can obtain.

VII. CONCLUSION

In this paper, we have presented a mathematical programming approach to derive optimal offers for a generation company operating in an electricity spot market consisting of a sequence of market mechanisms.

We have specified the market design assumed for our developments as well as the competition model embedded in our approach to clarify its merits and limitations. Our aim has been to reach a tradeoff between the modeling effort dedicated to the spot market and to the company's portfolio.

The mixed linear-integer mathematical programs that result when real study cases are addressed with this approach require the use of decomposition techniques for their solution. In this paper, we have explained a multicut version of Benders decomposition for this particular problem. A realistic numerical example in the context of the Spanish electricity market has been solved to illustrate its performance.

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