SHORT-TERM HYDROTHERMAL COORDINATION BY AUGMENTED LAGRANGEAN RELAXATION: A NEW MULTIPLIER UPDATING

Cesar Beltran, F.Javier Heredia
Statistics and Operations Research Department
Universitat Politècnica de Catalunya
Pau Gargallo 5, 08028-Barcelona. Spain
e-mail: heredia@eio.upc.es
Phone: +34-3-4017335
FAX: +34-3-4015855

Abstract : Augmented Lagrangean Relaxation Method (ALRM) is one of the most powerfull technique to solve the Short-Term Hydrothermal Coordination Problem (STHC Problem). A crucial step when using the ALR Method is the multipliers updating. In this paper we present an efficient new multiplier updating procedure: the Gradient Method with Radar Step. The method has been successfully tested solving large-scale exemples of the STHC Problem .

Keywords : Augmented Lagrangean Relaxation Method, Gradient Method with Radar Step, Large-scale Optimization, Short-term Hydro-thermal Coordination Problem, Variable Duplication.

Introduction

The problem we are going to deal with is called the Short-term Hydro-thermal Coordination (SHTC) Problem . The objective of the problem is the optimization of the electrical production and distribution considering a short-term planning horizon (from one day to one week). Hydraulic and thermal plants must be coordinated in order to reach the customer demand of electricity at the minimum cost and with a reliable service.

The model for the STHC Problem presented here considers the thermal park, the hydraulic park and the transmission network. The starting point will be the paper by Batut and Renaud [1992] and therefore we will use the Variable Duplication plus the Augmented Lagrangean Relaxation (ARL) Method. The improvement of Batut and Renaud Method will be theoretical and practical.Theoretically, one of the main drawbacks of the ALR Method is the multiplier updating because its heuristic character. We will introduce an effective and non heuristic updating procedure. From a practical point of view, an effective software designed to solve the Optimum Short-Term Hydrothermal Scheduling Problem (Heredia and Nabona [1995]) will be incorporated in order to speed up the whole algorithm.

This paper has been structured in the following sections:

1.- Formulation of the problem.
2.- Solution Algorithm: We report the main features of the ALR Method and introduce a new multiplier updating method, the Gradient Method with Radar Step.
3.- Modeling the STHC Problem : We report our particular model for the STHC Problem fully explained in Heredia and Nabona [1995].
4.- Solving the STHC Problem : The Gradient Method with Radar Step is implemented in the framework of the ALR Method in order to solve the STHC Problem .
5.- Computational tests.
6.- Conclusions.
7.- References.

Formulation

The optimization problems here considered are of the following type (P1):

$$\left.\begin{array}{rl} Min & f(x) = C_{D1}(x) + C_{D2}(x) \\ s.a. & x \in D_1 \\ & x \in D_2 \end{array}\right\} \qquad (1)$$

Where:

- D_1 represents the feasible set defined by the constraints coupling the hydro, thermal and transmission systems: load constraints, spinning reserve constraints, etc.
- D_2 represents the operating domain of the thermal units.

- $C_{D1}(x)$ represents the costs asociated with D_1
- $C_{D2}(x)$ represents the costs associated with D_2

We will use the method called Variable Duplication used already by the authors Batut and Renaud [1992]. The method of Variable duplication consists of exactly what the name proposes, duplicating the vector of the variables x giving way to $\tilde{x}$, to later add the constraint of equality i.e. $x = \tilde{x}$. Thus, we will solve the following transformation of (P1):

$$\left.\begin{array}{ll} Min & f(x,\tilde{x}) = C_{D1}(x) + C_{D2}(\tilde{x}) \\ s.a. & x \in D_1 \\ & \tilde{x} \in D_2 \\ & x = \tilde{x} \end{array}\right\} \qquad (2)$$

Solution algorithm.

The Lagrangean Relaxation Method is the most promising procedure to solve the STHC Problem . The initial Classical Lagrangean Relaxation Method was ameliorated by the ALR Method during the last decade. Recent advances in the multiplier updating (Cutting Plane Methods and Bundle Methods)for the Classical Lagrangean Relaxation have brought back to fashion this classical method. The multiplier method that we present in this paper improves notably the multiplier updating for ALR Method while keeps its previous advantages in relation to the Classical Lagrangean Relaxation Method.

Some advantages of the ALR Method are

- In the ALR Method we maximize a concave function: the dual function $q_c(\lambda)$.
- The ALR Method allows us to obtain a saddle-point even in the in cases where the Classical Lagrangean Relaxation Method presents a duality gap (Minoux [1983]). The solution of the STHC Problem by the Classical Lagrangean Relaxation Method usually yields an infeasible primal solution x_k due to the duality gap, whereas in the ALR Method a solution of the dual problem yields a primal solution.
- The ALR Method been a penalty method enjoys of its good performance characteristics and avoids its ill conditioning due to the need of large penalty parameters.
- Using the Classical Lagrangean Relaxation Method , the differentiability of the dual function cannot be ensured. Therefore subgradient methods must be applied in the Classical Lagrangean Relaxation Method . This difficulty can be overcome using an Augmented Lagrangean since the dual function q_c is differentiable for an appropriate c (Bertsekas [1995]). Thus, the multipliers can be updated using 'large steps'.

The weaknesses of the ALR Method are:

- The quadratic terms introduced by the Augmented Lagrangian are not separable. If we want to solve a problem by decomposition, some methods such as the Auxiliary Problem Principle, Cohen [1980], or the Block Coordinate Descend (Bertsekas [1995]) must be used. On the other hand the Classical Lagrangean Relaxation Method gives a separable Lagrangian.
- The multiplier updating is done in a heuristic way (Bertsekas [1995]) that needs to be tuned.

$$\lambda_{k+1} = \lambda_k + c_k \nabla q_c(\lambda_k) \qquad (3)$$

We will introduce a new multiplier updating procedure that overcomes completely this difficulty: the Gradient Method with Radar Step.

The Gradient Method with Radar Step.

The objective of the method is to maximize a differentiable and concave function $q(\lambda)$ without constrictions. This method uses the same information than the Cutting Plane Methods but in a different way. The tangent planes obtained in the course of the optimization give us a first order approximation of $q(\lambda)$. The Cutting Plane Method directly optimizes the successive approximations of $q(\lambda)$. The Radar Step Method uses the approximation to $q(\lambda)$ in order to compute the step length for an ascend direction (such as the gradient). Although convergence of the Radar Step method has not been proved yet, experience shows a very good behavior regarding convergence.

The Gradient-Radar Method can be summarized as follow:

GRM1.- Take an initial estimate λ_0 of the optimum. Set k= 0.
GRM2.- Compute the gradient vector $g_k := \nabla q(\lambda_k)$. Let Π_k be the first order approximation of $q(\lambda)$ at the point $(\lambda_k, q(\lambda_k))$ i.e. Π_k is the tangent plane defined by g_k . Store the tangent plane Π_k
GRM3.- If $g_k = 0$ then stop. λ_k is the optimum.
GRM4.- Computing the step length. Move on Π_k in such a way that λ_{k+1} follows the line $\lambda_{k+1} = \lambda_k + \beta g_k$, with $\beta > 0$. Keep moving up to the first stopping tangent plane Π_j with $j < k$, that means we stop the advance of λ_{k+1} for a value of β, lets call this value β_k. If no such stopping plane exist set $\beta_k = \frac{step}{\|g_k\|}$, for a prefixed value of *step*. Compute $\lambda_{k+1} = \lambda_k + \beta_k g_k$
GRM5.- Set $k = k + 1$ and go back to GRM2

Augmented Lagrangean plus Gradient-Radar Method.

The objective of the ALR Method is to maximize a differentiable and concave function (the dual function $q_c(\lambda)$) without restrictions, characteristics fully coincident with the requirements of the Gradient-Radar Method. Note that in the resolution of the STHC Problem we must relax only equality restrictions of the primal problem in order to get a dual problem with no restrictions upon the dual variables (multipliers). Then the algorithm that we will use to solve the STHC Problem is summarized as follow:

Suppose we want to solve (P3) $Min\{f(x) : x \in D, h(x) = 0\}$ then the ALR Method solves $Max\{q_c(\lambda) : \lambda \in R^n\}$ where

$$q_c(\lambda) := Min\{f(x) + \lambda' h(x) + c \parallel h(x) \parallel^2 : x \in D\} \tag{4}$$

$q_c(\lambda)$ is called the Dual Function, and

$$L_c(x, \lambda) = f(x) + \lambda' h(x) + c \parallel h(x) \parallel^2 \tag{5}$$

is called the Augmented Lagrangean Function. The ALR Method can be summarized in the following steps:

ALR1.- Take an initial estimate λ_0 of the Lagrange Multipliers. Set k= 0.
ALR2.- Compute $q_c(\lambda_k)$ to obtain x_k.
ALR3.- If the gradient of the dual function $\nabla q_c(\lambda_k) = 0$ then stop. x_k optimizes (P3).
ALR4.- Otherwise actualize the multipliers λ_k using the Gradient-Radar Method.
ALR5.- Set k= k+1 and go back to ALR2.

This new multiplier updating improves the classical updating in:

- No parameter tuning needs to be done.
- The Radar Step Method, unlike the classical multiplier updating method, is based on a direct knowledge of the dual function given that uses a first order approximation of the dual function.
- The information used by the Radar Step Method is free of any cost in the Lagrangean Framework because the gradient of the dual function is given by the unfeasibility of the relaxed constriction i.e. $\nabla q_c(\lambda_k) = h(x_k)$
- The consequence of this almost free knowledge of the dual function is a computationally efficient and faster updating method.

Modeling the STHC Problem

The general expression of the Short-Term Hydrothermal Coordination problem (P1) could be developed in several different ways. The approach adopted in this paper follows the so called *Coupled Model* presented in Heredia and Nabona [1995]. This model takes into account the hydroelectric energy generation system as well as the thermal system and the transmission network. The variable vector x of problem (P1) splits in three different vectors, x_H for the variables related with the hydroelectric system (volume, discharges and spillages of each reservoir), x_T for the thermal variables (power output and spinning reserve of each thermal unit), and variables x_E which accounts for the power flow through the electric transmission network. In the Coupled Model the constraints relating all these variables (domain D_1 of problem (P1)) are expressed through a network flow model with side constraints:

$$A_{HTT}\begin{pmatrix} x_H \\ x_T \\ x_E \end{pmatrix} = b_{HTT} \tag{6}$$

$$h(x_H, x_E) = 0 \tag{7}$$

$$T_{ISR} x_T \geq b_{ISR} \tag{8}$$

$$T_{DSR} x_T \geq b_{DSR} \tag{9}$$

$$T_{KVL} x_E = 0 \tag{10}$$

$$\underline{x}_H \leq x_H \leq \bar{x}_H \tag{11}$$

$$\underline{x}_T \leq x_T \leq \bar{x}_T \tag{12}$$

$$\underline{x}_E \leq x_E \leq \bar{x}_E \tag{13}$$

where

- (6): are the network constraints are those associated with the so called *Hydro-Thermal-Transmission Extended Network* (HTTEN). The HTTEN integrates the replicated hydro network, which accounts for the time and space coupling among the reservoirs of the river basin, the *thermal equivalent network* which defines the relation between the power output and the spinning reserve level of each thermal unit, and the transmission network, which formulates the conservation of the power flow at the busses of the transmission system.
- (7): these nonlinear side constraints defines the injection of the hydroelectric generation (a nonlinear function of the variables x_H) into the appropriate busses of the transmission network.
- (8),(9): These two sets of linear side constraints impose the satisfaction of the incremental and decremental spinning reserve requirements of the whole system.
- (10): these last set of linear side constraints are the formulation of the Kirchoff Voltage Law. These constraints, together with the power flow conservation equations formulated in (6), represents a dc approach to the transmission network.
- (11),(12),(13): upper and lower bounds to the variables.

The formulation of the domain D_1 as a network flow problem with side constraints allows the use of specialised network optimization codes. Also, the flexibility of this model is such that other relevant system constraints can be easily added, as, for instance, security constraints and emission constraints (Chiva et al. [1995]).

The operating domain D_2 of problem (P1) copes with the restrictions of the unit commitment problem, namely, the minimum down time and maximum up time of the thermal units.

The first term of the objective function of (P1), $C_{D1}(x_T)$ represents the cost of the fuel consumption of the thermal units, and it is modeled as a quadratic function of the power output of each thermal unit. This term could also include an estimation of the cost of the power losses through a quadratic function of some of the variables x_E. The second part, $C_{D2}(x)$ includes the start-up and shut-down costs of the thermal units, and depends only on the thermal variables x_T.

Solving the Short-Term Hydrothermal Coordination Problem: the MACH algorithm.

We will follow and improve the method described by Batut and Renaud [1992] in the solution of the STHC Problem . The method uses Augmented Lagrangean Relaxation and Duplication of variables, previous software used to solve the Dispatching Problem and the Optimal Power Flow can be incorporated and the Augmented Lagrangean losses its separability.

The algorithm we will use falls in the class of ALR Method but it enjoys of a better multiplier updating than the classical versions. The non-separability of the Lagrangean is overcome using the Block Coordinated Descent Method.

Let's suppose we have the following information available : an initial estimate of the Lagrange multipliers λ_0; a penaly parameter c; a positive integer K, which serves as an upper bound to the number of iterations of the Block Coordinated Descend at each minimization of the Aumented Lagrangian; a positive integer N, which serves as an upper bound to the number of Lagrange multiplier updates; an initial point x_0 of the domain D_1 and an initial point $\tilde{x}_0$ of the domain D_2. Let $k = 0$ y $n = 0$. Then the algorithm proposed, called MACH (from *"Modelo Acoplado de Coordinación Hidrotérmica"*) will be:

MACH1.- [Test the terminating criteria] If x_k and $\tilde{x}_k$ satisfies the conditions of optimality, the algorithm finishes with $(x_k, \tilde{x}_k)$ as a solution . If $n > N$, the algorithm has failed.

MACH2.- [Minimize the augmented Lagrangian in D_1] With x_k as an initial point and $\tilde{x}_k$ as a fixed vector, execute a procedure to solve the following subproblem:

$$\min_{x \in D_1} \; L_c(x) \quad = \quad L_c(x, \tilde{x}_k, \lambda_k) \tag{14}$$

including measures of security to cope with unboundedness. Let x_{k+1} be the calculated solution.

MACH3.- [Minimize the augmented Lagrangian in D_2] With $\tilde{x}_k$ as an initial point and x_{k+1} as a fixed vector, execute a procedure to solve the following subproblem:

$$\min_{\tilde{x} \in D_2} \; L_c(\tilde{x}) \quad = \quad L_c(x_{k+1}, \tilde{x}, \lambda_k) \tag{15}$$

including measures of security to cope with unboundedness. Let $\tilde{x}_{k+1}$ be the calculated solution.

MACH4.- [Repeat steps MACH2 and MACH3 until no progress can be done] If $k \geq K$ then go to MACH5) Else if

$$\| \, x_{k+1} - x_k \, \| \; > \; \epsilon \qquad o \qquad \| \, \tilde{x}_{k+1} - \tilde{x}_k \, \| \; > \; \epsilon \tag{16}$$

Set $k = k + 1$ and go back to step MACH1.

MACH5.- [Update the multiplier estimates using the Gradient-radar Step]

$$\lambda_{n+1} = \lambda_n + \beta(x_{k+1} - \tilde{x}_{k+1}) \tag{17}$$

MACH6.- [Update the λ iteration count] Set $n = n + 1$, $k = k + 1$ and go back to step MACH1.

One of the advantages of the Duplication Variable framework is the possibility of incorporating preexisting software. In step **MACH2** of the MACH algorithm the minimization of the Augmented Lagrangean subject to the constraints (6) to (13) is needed. This is a nonlinear network flow problem with side constraints which can be solved either with general purpose optimization packages or with specialised procedures. The implementation reported in this paper is based on the specialised code NOXCB Heredia and Nabona [1992]. This code implements an active set method which exploits the network structure through primal partitioning techniques (Kennington and Helgason [1982]) to solve the nonlinear network problem with linear side constraints. To handle the nonlinear constraint (7) a successive linearization method presented in Heredia and Nabona [1995] is used. In this method, a sequence of subproblems are solved. In these subproblems the nonlinear constraints (7) are linearized over the optimal solution of the previous subproblem. The linearizations stops when a given convergence criterion is reached. Furthermore, this framework will allow in the future to incorporate new packages, as for example Interior Point based software to solve step **MACH2**.

In step **MACH3** a classical Dynamical Programming Procedure has been implemented. The characteristics of the subproblem (15) (binary variables plus separability), in our opinion, makes the Dynamical Programming Procedure one of the best options.

A second major feature of the Duplication Variable framework is the possibility of incorporating new constrictions. The Lagrangean Relaxation performed is independent of the system constrictions (demand, spinning reserve, etc.) and therefore new constrictions such as pollution regulations can be easily added with no need of adding a new set of Lagrangean multipliers.

Computational tests.

So far we have tested the method considering the Hydraulic and Thermal Systems without Distribution Network, although the software developed can incorporate the Distributin Network. The problems solved fall in the medium and large-scale size. From 24 to 168 hours, about 10 thermal units and 10 hydroelectric stations. After our experience with the MACH package, the main conclusions are: (1) Usually the method reaches the optimum within 20 or less multiplier updating, in front of the 100 or more needed by the classical multiplier updating. (2) Usually there is no dual gap and the primal solution obtained is ready to be used, unlike the Classical Lagrangean Relaxation Method.

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