Stochastic optimal day-ahead bid with physical future contracts

C. Corchero, F.J. Heredia
Departament d'Estadística i Investigació Operativa
Universitat Politècnica de Catalunya

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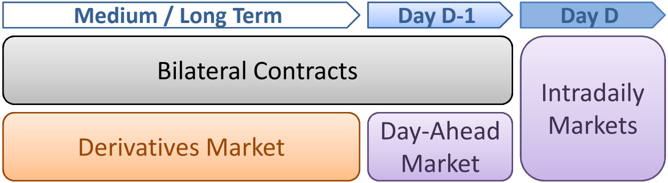
Electric Energy Iberian Market: MIBEL



Derivatives Market

Physical Futures Contracts
Financial and Physical Settlement. Positions are sent to OMEL's Mercado Diario for physical delivery.
Financial Futures Contracts
OMIClear cash settles the differences between the Spot Reference Price and the Final Settlement Price

Bilateral Contracts

Organized markets
- Virtual Power Plants auctions (**EPE**) - Distribution auctions (**SD**) - International Capacity Interconnection auctions - International Capacity Interconnection nomination
Non organized markets
- **National** BC **before** the spot market - **International** BC **before** the spot market - **National** BC **after** the spot market

Day-Ahead Market

Day-Ahead Market
Hourly action. The matching procedure takes place 24h before the delivery period. Physical futures contracts are settled through a zero price bid.

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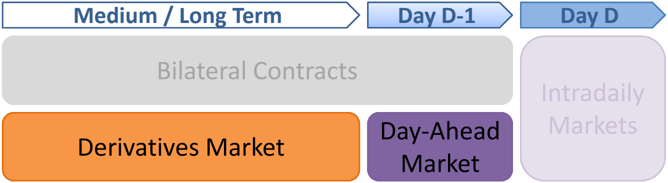

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Characteristics of Physical Futures Contracts

Main characteristics

- Base load
- Physical or financial settlement.
- Delivery period: years, quarters, months and weeks.

Definition

- A *Base Load Futures Contract* consists in a pair (L^f, λ^f)
  - L^f: amount of energy (MWh) to be procured each interval of the delivery period.
  - λ^f: price of the contract (c€/MWh).

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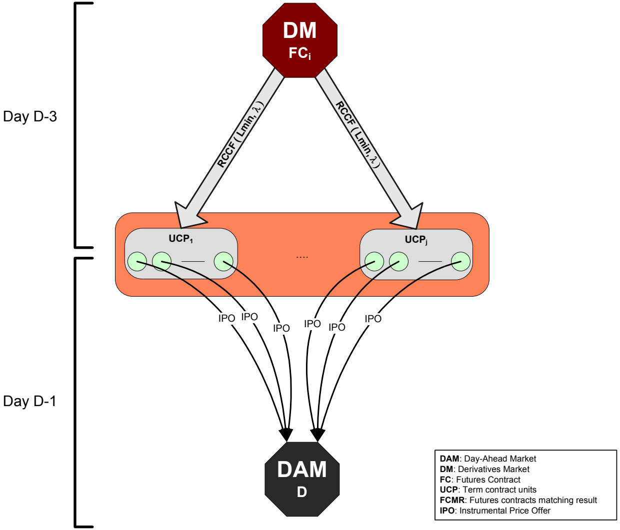
Physical Futures Contracts and Day Ahead Market



Problem definition

The objective of the study is to decide:

- the optimal economic dispatch of the physical futures contract among the thermal units
- the optimal bidding at Day-Ahead Market abiding by the MIBEL rules
- the optimal unit commitment of the thermal units

maximizing the expected Day-Ahead Market profits taking into account futures contracts.

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Optimal bid curve **without** future contracts (I/II)

For a given spot price λ_i, the benefit function of the *committed* unit t is:

$$B_i^t(p_i^t) = \lambda_i p_i^t - \left(c_b^t + c_l^t p_i^t + c_q^t (p_i^t)^2\right) \ , \ p_i^t \in [\underline{P}^t, \overline{P}^t] \qquad (1)$$

and the generation $p_i^{d,t}$ that maximizes $B_i^t(p_i^t)$ is:

$$p_i^{d,t}(\lambda_i) = \begin{cases} \underline{P}^t & \text{if} \quad p_i^{*t}(\lambda_i) \le \underline{P}^t \\ \overline{P}^t & \text{if} \quad p_i^{*t}(\lambda_i) \ge \overline{P}^t \\ p_i^{*t}(\lambda_i) & \text{otherwise} \end{cases} \qquad (2)$$

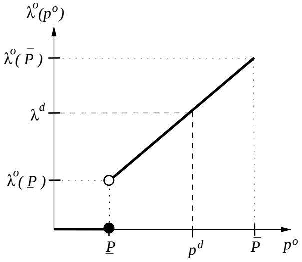
where $p_i^{*t}(\lambda_i) = \left(\lambda_i - c_l^t\right)/2c_q^t$ is the unconstrained maximum of the benefit function (1)

Optimal bid curve **without** future contracts (II/II)

The day-ahead optimal bid curve $\lambda_i^{o,t}(p_i^{o,t})$ that maximizes the benefit function (1) for any given spot price λ_i is the expression derived from (2) :

$$\lambda_i^{o,t}(p_i^{o,t}) = \begin{cases} 0 & \text{if} \quad 0 \leq p_i^{o,t} \leq \underline{P}^t \\ 2c_q^t p_i^{o,t} + c_l^t & \text{if} \quad \underline{P}^t < p_i^{o,t} \leq \overline{P}^t \end{cases} \qquad (3)$$

graphically:



Optimal bid curve **with** future contracts (I/II)

- Let q_i^t be the generation of thermal t at time i allocated to all the physical contracts of the portfolio.
- The market rules forces each generator to send the amount q_i^t to the Day-Ahead Market through an instrumental price bid (bid at zero price).
- For a given value q_i^t, the optimal bid curve is the function $\lambda_i^{o,t}(p_i^{o,t}; q_i^t)$ that provides the energy-price pairs $(p_i^{o,t}, \lambda_i^{o,t})$ that maximize the benefit function for any given spot price λ_i.

Optimal bid curve **with** future contracts (II/II)

The expression of the optimal bid curve for thermal unit t at time interval i, for a given q_i^t, is:

$$\lambda_i^{o,t}(p_i^{o,t}; q_i^t) = \begin{cases} 0 & \text{if} \quad 0 \leq p_i^{o,t} \leq q_i^t \\ 2c_q^t p_i^{o,t} + c_l^t & \text{if} \quad q_i^t < p_i^{o,t} \leq \overline{P}^t \end{cases} \tag{4}$$

graphically:

pgflastimage

Matched energy

Given a spot price λ_i^s, corresponding to scenario s, and a value q_i^t, the matched energy p_i^{ts} is completely determined through expression (4), and depends on the comparison between q_i^t and p^{ts}:

$$p_i^{ts} = \begin{cases} q_i^t & \text{if } q_i^t \geq p_i^{d,ts} \\ p_i^{d,ts} & \text{otherwise} \end{cases} \tag{5}$$

where the constant $p_i^{d,ts}$ is the generation that maximizes the benefit function for a given spot-price λ_i^s (2).

Problem definition

Model characteristics

- Stochastic mixed integer quadratic programming model
- *Price-taker* generation company
- Set of thermal generation units, T
- Optimization horizon of 24h, I
- Set of physical futures contracts, F
- Set of day-ahead market price scenarios, $\lambda^s \in \Re^{|I|}$, $s \in \mathcal{S}$

Variables

First stage variables: $\forall t \in T, \forall i \in I$

- Unit commitment: $u_i^t,\ a_i^t,\ e_i^t \in \{0,1\}$
- Instrumental price offer bid : q_i^t
- Scheduled energy for contract j: $f_{ij}^t \quad \forall j \in F$

Second stage variables $\forall t \in T, \forall i \in I, \forall s \in S$

- Matched energy: p_i^{ts}

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Physical Future Contracts constraints

Physical future contract covering:

$$\sum_{t \in T} f_{ij}^{t} = L_j \, , \forall j \in F$$

Instrumental price bid:

$$q_i^t \geq \sum_{j \in F} f_{ij}^{t} \, , \forall t \in T \, , \forall i \in I$$

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System constraints

Start-up/Shut-down constraints: $\forall i \in I$, $\forall t \in T$

$$u_i^t - u_{i-1}^t - e_i^t + a_i^t = 0$$

$$a_i^t + \sum_{k=i+1}^{min\{i+tm_t^{off},|I|\}} e_j^t \leq 1$$

$$e_i^t + \sum_{k=i}^{min\{i+tm_t^{on},|I|\}} a_k^t \leq 1$$

Operational constraints: $\forall i \in I$, $\forall t \in T$, $\forall s \in S$

$$p_i^{ts} \in 0 \cup [\underline{P}^t, \overline{P}^t]$$

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$$f_{ij}^t \geq 0$$

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Objective function

$$\min_{p,q,f,u,a,e} \sum_{\forall i \in I} \sum_{\forall t \in T} c_{on}^t e_i^t + c_{off}^t a_i^t + c_b^t u_i^t + \sum_{s \in S} P^s \left[(c_l^t - \lambda_i^s) p_i^{ts} + c_q^t (p_i^{ts})^2 \right]$$

Coherency of the model with the optimal bidding curve

It can be proved that at every solution of the Karush-Kuhn-Tucker system the value of the primal variables p_i^{ts} and q_i^t satisfies the same relation than the matched energy

$$p_i^{ts} = \begin{cases} q_i^t & \text{if } q_i^t \geq p_i^{d,ts} \\ p_i^{d,ts} & \text{otherwise} \end{cases} \tag{6}$$

where

$$p_i^{d,ts}(\lambda_i^s) = \begin{cases} \underline{P}^t & \text{if} \quad p_i^{*t}(\lambda_i) \leq \underline{P}^t \\ \overline{P}^t & \text{if} \quad p_i^{*t}(\lambda_i) \geq \overline{P}^t \\ \left(\lambda_i^s - c_l^t\right)/2c_q^t & \text{otherwise} \end{cases} \tag{7}$$

Case Study characteristics

- Real data from the Spanish Market about the generation company and the market prices.
- 10 thermal generation units (7 coal, 3 fuel) from a Spanish generation company with daily bidding in the MIBEL

$[\overline{P} - \underline{P}]$ (MW)	160-243	250-550	80-260	160-340	30-70
$min_{on/off}$ (h)	3	3	3	4	4
$[\overline{P} - \underline{P}]$ (MW)	60-140	160-340	90-340	110-157	110-157
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- Model implemented and solved with AMPL/CPLEX 10.0.
- CPU time using a SunFire V20Z with two processors AMD Opteron at 2.46Hz and 8Gb of RAM memory.

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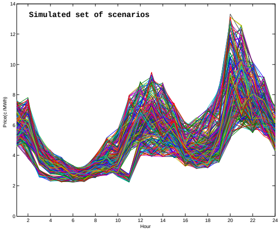
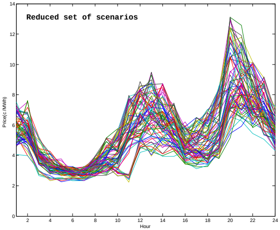
Stochasticity modeling

- Price Spot Market, $\lambda_i^{d,s}$, is characterized as a time series
- Time series study results in a ARIMA model:

$$\text{ARIMA } (23,1,13)(14,1,21)_{24}(0,1,1)_{168}$$

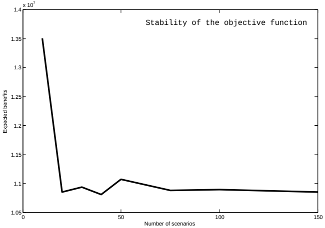
- Price scenario construction:
  - Generation of 350 scenarios by time series simulation
  - Reduction of the number of scenarios [1]

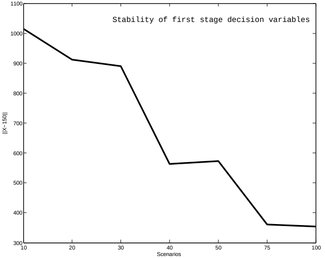




[1] Gröwe-Kuska et al. *Scenario Reduction and Scenario Tree Construction for Power Management Problems*

Stability analysis

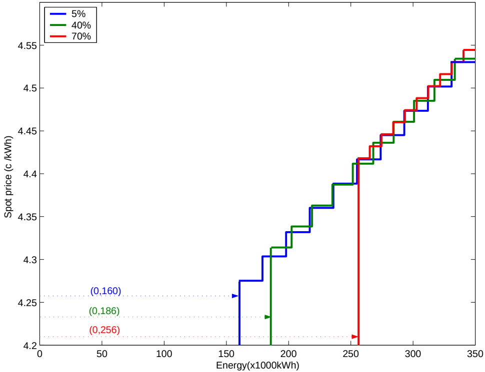




$\|S\|$	c.v.	CPU(s)	E(benefits)(€)	$\|\|\Delta(€)/\Delta(s)\|\|$
10	3.360	13	1.350.830	
20	5.760	55	1.085.240	6.323,57
30	8.160	112	1.093.900	151,93
40	10.560	216	1.081.010	123,94
50	12.960	444	1.107.110	114,47
75	**18.960**	**2.100**	**1.087.860**	**11,62**
100	24.960	3.319	1.089.280	1,16
150	36.960	4.244	1.084.880	4,76

$|I| = 24$; $|T| = 10$; $\%\overline{P} = 40$; b.v.$= 720$

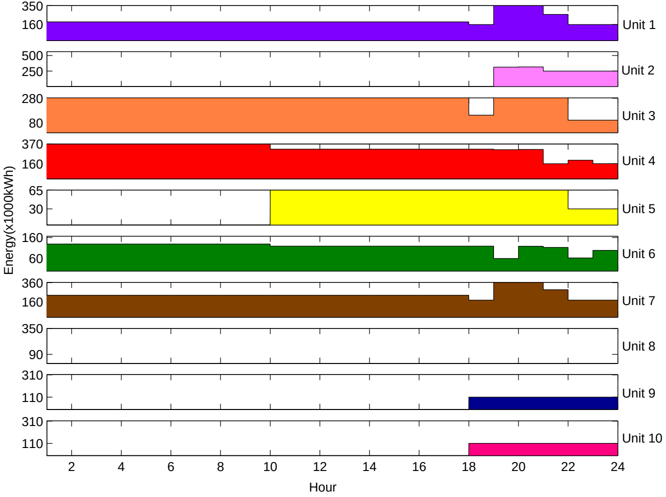
Optimal bidding strategy by futures contracts quantity



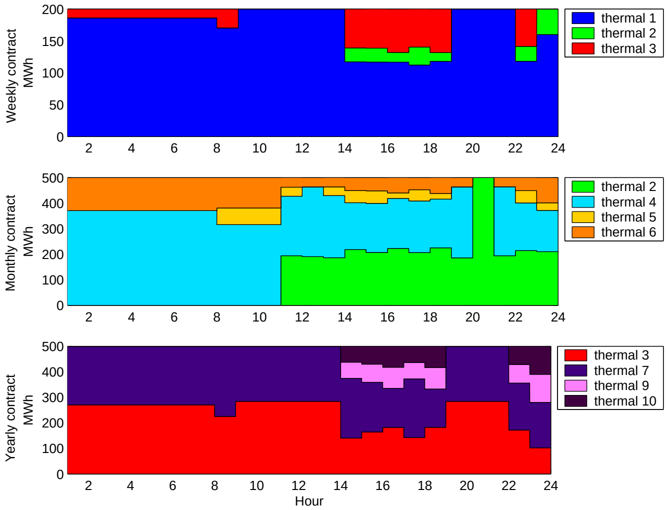
$\%\overline{P}$	E(benefits)
5	1.823.170
40	1.107.110
75	-2.800.460

$|I| = 24$; $|T| = 10$; $|S| = 75$;
c.v. $= 720$; b.v. $= 12960$

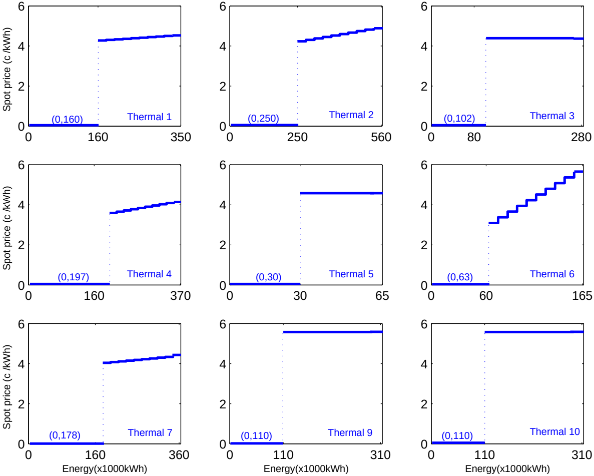
Results: unit commitment and zero price bid



Results: procurement of physical futures contracts



Results: optimal bidding curves



Conclusions

- It has been built an Optimal Bidding Model for a price-taker generation company operating both in the MIBEL Derivatives and Day-Ahead Electricity Market.
- The stochasticity of the spot market price has been taken into account and it has been represented by a scenario set.
- The model developed gives the producer:
  - Optimal bid for the spot market: quantity at 0€/MWh and the rest of the power capacity at the unit's marginal cost
  - Unit commitment
  - Optimal allocation of the physical futures contracts among the thermal units

following in detail the MIBEL rules.

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