Electricity Market Optimization: finding the best bid through stochastic programming.

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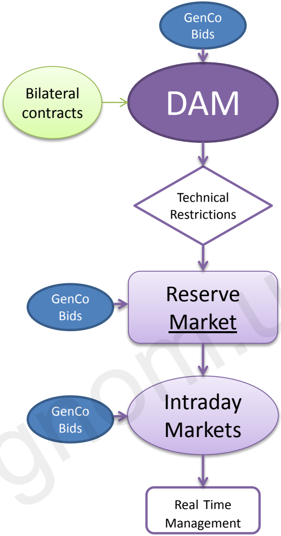
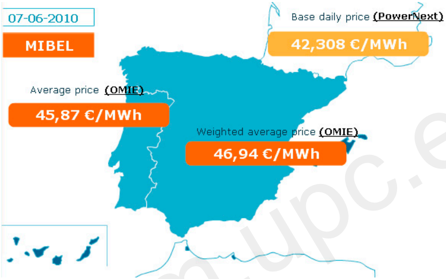
1. The problem : Iberian Electricity Market
   - Iberian Electricity Market (MIBEL)
   - Day-Ahead Market (DAM) in the MIBEL
   - GenCo's optimal DAM bid problem
2. The model: stochastic programming
3. The optimization: perspective cuts
4. The results: from data to optimal bid.
5. Conclusions

Spanish Electricity Market



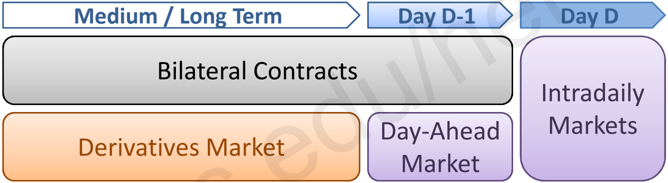
- The Spanish Electricity Market started up at January 1998.
- It stablished a fully competitive framework for the generation of electricity, with a set of market mechanism centralized and managed by the *market operator*.
- It included a Day Ahead Market, a Reserve Market and a set of Intraday Markets to which the generation companies (GenCo) could submit their sell bids.

Iberian Electricity Market (MIBEL)



- The MIBEL (created in 2007) joins Spanish and Portuguese electricity system and it complements the previous mechanisms of the Spanish Electricity Market with a Derivatives Market.
- This Derivatives Market has its own market operator called *OMIP*, located in Portugal (`www.omip.pt`)
- The old Spanish market operator is renamed as *OMIE* and it is still in charge of the spot markets (`www.omel.es`).

Markets in the MIBEL



Derivatives Market

Physical Futures Contracts
Financial and Physical Settlement. Positions are sent to OMEL's Mercado Diario for physical delivery.
Financial Futures Contracts
OMIClear cash settles the differences between the Spot Reference Price and the Final Settlement Price

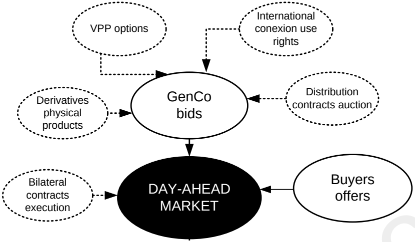
Bilateral Contracts

Organized markets
- Virtual Power Plants auctions (**EPE**) - Distribution auctions (**SD**) - International Capacity Interconnection auctions - International Capacity Interconnection nomination
Non organized markets
- **National** BC **before** the spot market - **International** BC **before** the spot market - **National** BC **after** the spot market

Day-Ahead Market

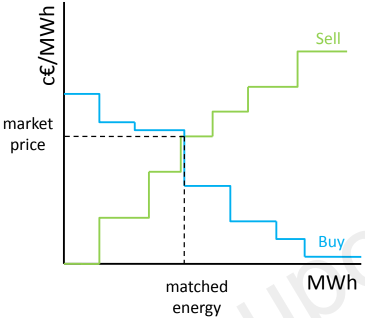
Day-Ahead Market
Hourly action. The matching procedure takes place 24h before the delivery period. Physical futures contracts are settled through a zero price bid.

Day-Ahead Market mechanism



- The Day-Ahead Market (DAM) is the most important part of the electricity market (78% of the total system demand traded through DAM on 2009).
- The objective of this market is to carry out the energy transactions for the next day by means of the selling and buying offers presented by the market agents.
- The DAM is formed up by twenty-four hourly auctions that are cleared simultaneously between 10:00 and 10:30am of day D-1.

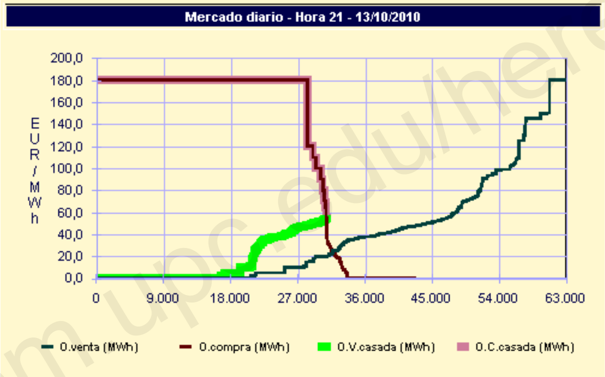
Clearing mechanism



- To derive the aggregate offer (demand) curve, sell (buy) bids are sorted by increasing (decreasing) prices and their quantities are accumulated.

- At each auction t the clearing-price λ_t is determined by the intersection of the aggregated supply and demand curve
- All the sale (purchase) bids with a lower (greater) bid price are matched and will be remunerated at the same clearing price λ_t irrespective of the original bid price.

Actual clearing for October 13th at 21h



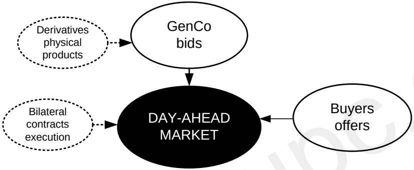
Clearing price : $\lambda_{21} = 54.01€/\text{MWh}$

Total energy traded : 30.816,2 MWh

(Source: `www.omel.es`)

The GenCo's optimal DAM bid problem

The GenCo's optimal DAM bid problem considers a *Price-Taker* generation company with:



- A set of thermal generation units, I, with quadratic generation costs, start-up and shut-down costs and minimum operation and idle times.
- Each generation unit can submit sell bids to the 24 auctions of the DAM.
- A set of physical futures contracts, F, of energy L_j^{FC} $j \in F$.
- A pool of bilateral contracts B of energy L_k^{BC}, $k \in B$.

Objectives

The objective of the study is to decide:

- the **optimal economic dispatch of the physical futures and bilateral contract** among the thermal units
- the **optimal bidding at Day-Ahead Market** abiding by the MIBEL rules
- the **optimal unit commitment** (binary on/off state) of the thermal units

maximizing the expected Day-Ahead Market profits taking into account futures and bilateral contracts.

Stochastic Programming

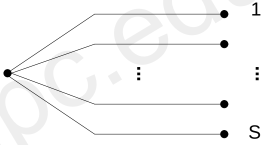
two-stage stochastic program with fixed recourse

$$
\begin{aligned}
min \quad & z = c^T x + E_\xi[min\ q(\omega)^T y(\omega)] \\
s.t. \quad & Ax = b \\
& T(\omega)x + Wy(\omega) = h(\omega) \\
& x \geq 0
\end{aligned}
$$

- x: *first-stage decision variables*
- c, b, A: first-stage vectors and matrices.
- $y(\omega)$: *second-stage decision variables* for a given realization of the random variable $\omega \in \Omega$
- $q(\omega)$, $h(\omega)$, $T(\omega)$: second stage data.

Scenarios

- In stochastic programming the probability distribution of the random variable $\omega \in \Omega$ is approximated through a discrete distribution with finite support (*set of scenarios*).



- The random data in this work are the prices $\lambda^s \in \Re^{|T|}$ at which the energy will be remunerated in the DAM.
- This random variables will be modeled through a set of scenarios S with associated spot prices λ^s and probabilities $P^s = P(\lambda^s)$, $s \in S$.

2 The model: stochastic programming

- Stochastic Programming
- Scenario generation
- Optimization model
- Optimal Bid Function

Outline

Hourly electricity prices λ (www.omel.es)

$\downarrow$

Stochastic modeling of λ

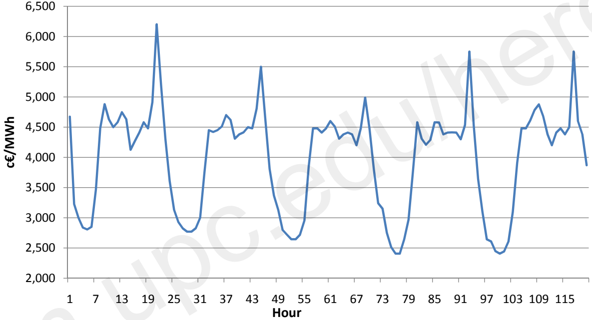
$\Downarrow$

Scenario generation

$\Downarrow$

Stochastic optimization model

Price characteristics



Electricity spot prices exhibit:

- Non-constant mean and variance
- Daily and weekly seasonality
- Calendar effects
- High volatility and presence of outliers

Approaches to the modeling of λ

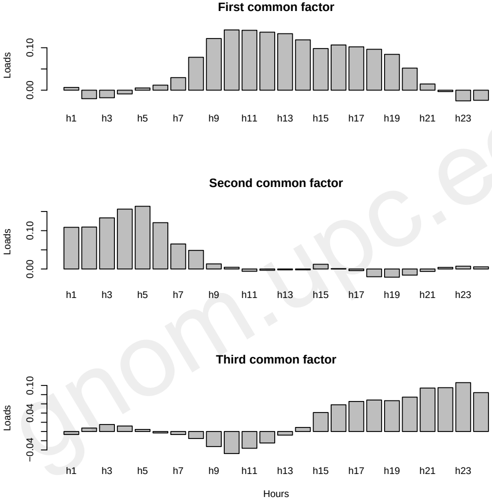
Several parametric and non-parametric approaches has been proposed:

- Non-parametric statistic methods - such as clustering or bootstrapping - applied to historical data.
- ARIMA models
- Neural networks models
- Dynamic regressions

Factor Model Approach (Muñoz, Corchero 2009)

- Time Series Factor Analysis (Gilbert P.D., Meijer E.2005) estimates measurement model for time series data with as few assumptions as possible about the dynamic process governing the factors. It estimates parameters and predicts factor scores.
- The forecasting model provide suitable scenarios for the optimization model.
- The factor model allows to identify common unobserved factors which represent the relationship between the hours of a day.

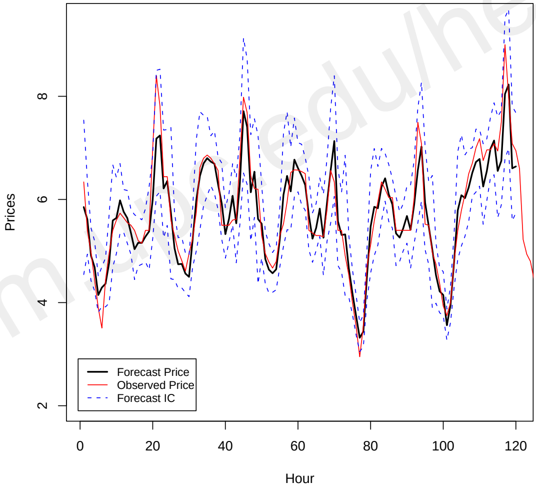
Factor model results



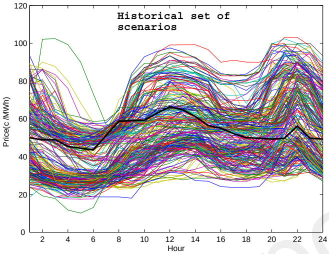
Loads of the common factors

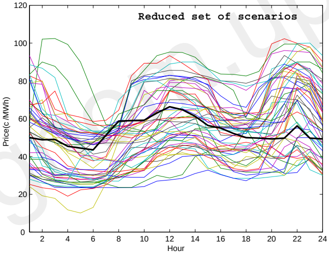
- 3 significant factors, based on eigenvalues of the sample correlation matrix.
- Data set: work days from January 1^{rts}, 2007 to January 1^{rts}, 2009.

Out of sample forecasting results



The spot price scenarios





An initial set of scenarios for the random variable λ is generated from the discretization of the confidence interval of the TSFA forecast (250 scenarios in the example).

Scenario reduction techniques are applied to reduce the number of scenarios preserving at maximum the characteristics of the observed data (50 scenarios in the example). [a]

[a] Gröwe-Kuska et al. Scenario Reduction and Scenario Tree Construction for Pow

2 The model: stochastic programming

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- Scenario generation
- Optimization model
- Optimal Bid Function

Variables

First stage variables: $t \in T, \ i \in I$

- Unit commitment: $u_{it} \in \{0,1\}$, c^u_{it}, c^d_{it}
- Instrumental price offer bid : q_{it}
- Scheduled energy for futures contract j: $f_{itj} \ \ j \in F$
- Scheduled energy for bilaterals contract: b_{it}

Second stage variables $t \in T, \ i \in I, \ s \in S$

- Matched energy: $p^{M,s}_{it}$
- Total generation: p^s_{it}

Physical Future Contracts modeling

- A *Base Load Futures Contract* $j \in F$ consists in a pair $(L_j^{FC}, \lambda_j^{FC})$
- L_j^{FC}: amount of energy (MWh) to be procured each interval of the delivery period by the set U_j of generation units.
- λ_j^{FC}: price of the contract (c€/MWh).

Physical future contract contraints:

$$\sum_{i \in U_j} f_{itj} = L_j^{FC} \, , \, j \in F \, , \, t \in T$$

$$f_{itj} \geq 0 \, , \, j \in F \, , \, i \in I \, , \, t \in T$$

Base Load Bilateral Contracts modeling

- A *Bilateral Contract* $k \in B$ consists in a pair $(L_k^{BC}, \lambda_k^{BC})$
- L_k^{BC}: amount of energy (MWh) to be procured each interval t of the delivery period.
- λ_k^{BC}: price of the contract (c€/MWh).

Bilateral contract constraints:

$$\sum_{i \in I} b_{it} = \sum_{k \in B} L_k^{BC} \ , \ t \in T$$

$$0 \leq b_{it} \leq \overline{P}_i u_{it} \ , \ i \in I \ , \ t \in T$$

MIBEL's bidding rules

(BR1) To guarantee its inclusion in the operational programming, any committed unit i ($u_{it} = 1$) would bid its minimum generation level $\underline{P}_i$ at zero price (instrumental price bid q_{it}).

(BR2) If generator i contributes with f_{itj} MWh at period t to the coverage of the FC j, then the energy f_{itj} must be included into the instrumental price bid q_{it}.

(BR3) If generator i contributes with b_{it} MWh at period t to the coverage of the BCs, then the energy b_{it} must be excluded from the bid to the DAM. Unit i can offer its remaining production capacity $\overline{P}_i - b_{it}$ to the pool.

Day-ahead market bidding model: constraints

Matched energy:

$$\text{(BR3)}\quad p_{it}^{M,s} \leq \overline{P}_i u_{it} - b_{it}\ ,\ i \in I\ ,\ t \in T\ ,\ s \in S$$

$$\text{(BR1)}\quad p_{it}^{M,s} \geq q_{it}\ ,\ i \in I\ ,\ t \in T\ ,\ s \in S$$

Instrumental price bid:

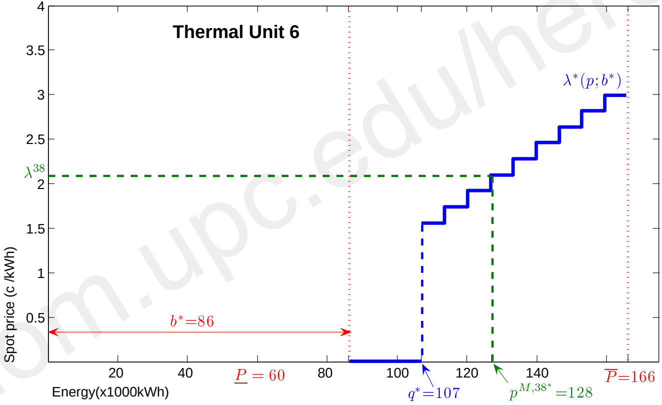
$$\text{(BR1+3)}\quad q_{it} \geq \underline{P}_i u_{it} - b_{it}\ ,\ i \in I\ ,\ t \in T$$

$$\text{(BR2)}\quad q_{it} \geq \sum_{j|i \in U_j} f_{itj}\ ,\ t \in T\ ,\ i \in I$$

Total energy generation:

$$p_{it}^{s} = b_{it} + p_{it}^{M,s}\ ,\ t \in T\ ,\ i \in I\ ,\ s \in S$$

Graphical representation of the optimal bidding curve



Optimal bidding curve for thermal unit 6 at interval 18

Day-ahead market bidding model: assumption

Our optimization model doesn't include any explicit representation of the optimal bid function. It relies on the following assumption:

Assumption

For any thermal unit i committed at period t there exists a bid function (optimal) λ^b_{it} such that:

$$p^{M,s*}_{it} = p^M_{it}(\lambda^s_t) \quad \forall s \in S \qquad (1)$$

with $p^{M,s*}_{it}$ the optimal value of the matched energy variable $p^{M,s}_{it}$

The correctness of this assumption can be proved.

Start-up and shut-down costs

- Let c_i^{on} and c_i^{off} be the constant start-up and shut-down costs, respectively, of thermal unit i.

Start-up and shut-down costs constraints:

$$c_{it}^{u} \geq c_i^{on}[u_{it} - u_{i,(t-1)}] \qquad t \in T \setminus \{1\}\ ,\ i \in I$$

$$c_{it}^{d} \geq c_i^{off}[u_{i,(t-1)} - u_{it}] \qquad t \in T \setminus \{1\}\ ,\ i \in I$$

Minimum operation and idle time

- Let t_i^{on} (resp. t_i^{off}) be the number of consecutive periods unit i must be online (offline) once it has been turned on (shut down).
- There is a set of complicated linear inequalities involving variables u_{it} and parameters t_i^{on}, t_i^{off}, that ensures satisfaction of the minimum up/down times of each unit i.

Minimum up/down time constraints:

$$u \in X(t^{on}, t^{off})$$

Objective function

Maximization of the day-ahead market clearing's benefits

$$\max_{p,q,f,b} \sum_{t\in T} \sum_{i\in I} \Big(- c_{it}^{u} - c_{it}^{d} - c_{it}^{b} u_{it} + \\ + \sum_{s\in S} P^s \left[\lambda_t^{Ds} p_{it}^{M,s} - (c_i^l p_{it}^s + c_i^q (p_{it}^s)^2) \right] \Big)$$

Incomes from Futures and bilateral contracts (constant):

- **Futures contracts:** $\sum_{t\in T} \sum_{j\in J} \left(\lambda_j^{FC} - \lambda_t \right) L_t^{FC}$
- **Bilateral contracts:** $|T| \sum_{k\in B} \lambda_k^{BC} L_k^{BC}$

Summary of the model

Problem DABFC
(**D**ay-**A**head bid with **B**ilateral and **F**utures **C**ontracts)

Max E[Profit from the Day-ahead market]

s.t. Physical future contract coverage

Bilateral contract coverage

Matched energy

Instrumental price bid

Total energy generation

Start-up and shut-down costs

Minimum operation and idle time

(Mixed Integer Quadratic Programming (MIQP) problem)

2 The model: stochastic programming

- Stochastic Programming
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- Optimal Bid Function

Day-ahead market bidding model: assumption

Remember that the DABFC model was built assuming the following:

Assumption

For any thermal unit i committed at period t there exists a bid function (optimal) λ^b_{it} such that:

$$p^{M,s*}_{it} = p^M_{it}(\lambda^s_t) \quad \forall s \in S \tag{2}$$

with $p^{M,s*}_{it}$ the optimal value of the matched energy variable $p^{M,s}_{it}$

DABFC's optimal bid function

The following bid function can be proved to match the assumption:

Lemma (Optimal bid function)

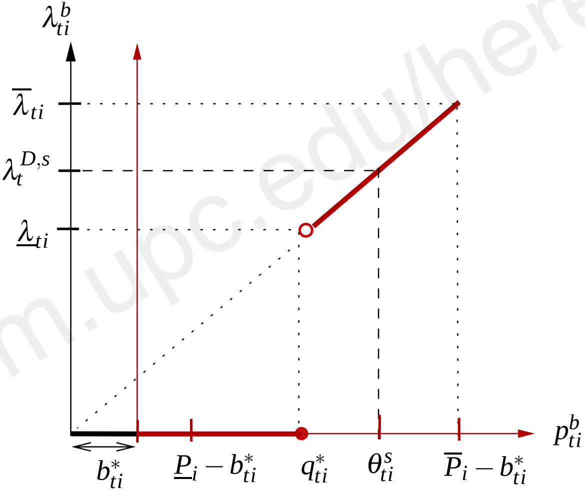
Let $x^{*\prime} = [p^{M,*}, p^*, q^*, f^*, b^*, u^*]'$ be an optimal solution of the (DABFC) problem and i any thermal unit committed on period t at the optimal solution. Then the bid function:

$$\lambda^*_{it}(p_{it}; b^*_{it}, q^*_{it}) = \begin{cases} 0 & \text{if } p_{it} \leq q^*_{it} \\ 2c^q_i\left(p_{it} + b^*_{it}\right) + c^l_i & \text{if } q^*_{it} < p_{it} \leq (\overline{P}_i - b^*_{it}) \end{cases}$$

is optimal w.r.t. the (DABFC) problem and the optimum x^*.

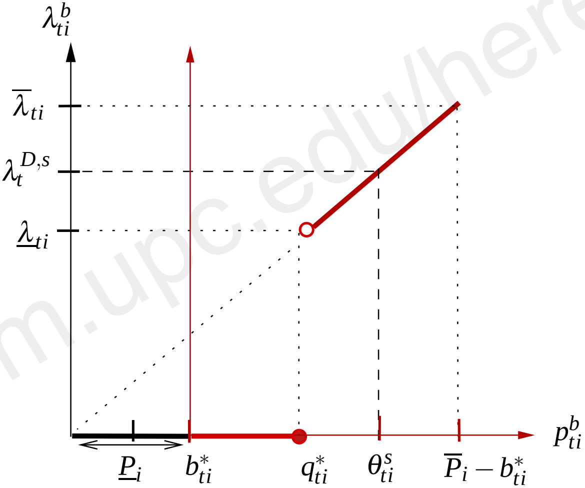
(Proof: KKT conditions of DABFC)

DABFC's optimal bid function



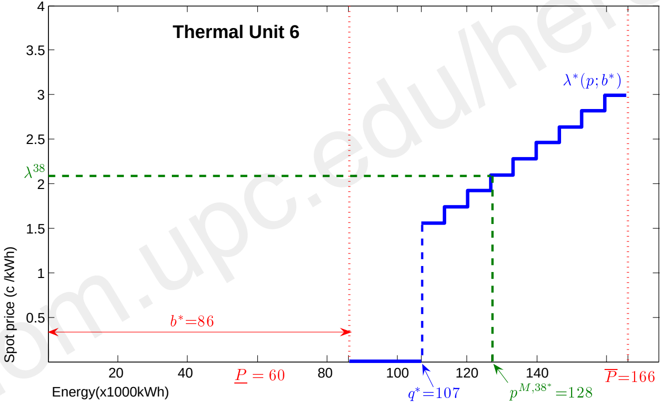
(a)

DABFC's optimal bid function



(b)

Actual DABFC's optimal bid



Optimal bidding curve for thermal unit 6 at interval 18

1. The problem : Iberian Electricity Market
2. The model: stochastic programming
3. The optimization: perspective cuts
   - Convex envelope and the perspective function
   - Implementation and numerical tests
4. The results: from data to optimal bid.
5. Conclusions

Motivation

- Model (DABFC) is a Mixed-Integer Quadratic Program (MIQP), which is difficult to solve efficiently, especially for large-scale instances.
- A possibility is to approximate the quadratic objective function $f(p,u)$

$$f(p,u) = c^q p^2 + c^l p + c^b u$$

by means of perspective cuts (Frangioni and Gentile, 2006), so that this problem can be solved as a Mixed-Integer Linear Program (MILP) by general-purpose MILP solvers.

PC formulation (PCF)

Objective function for PCF

$$\min_{p,q,f,b} \mathrm{E}_{\lambda^{\mathcal{D}}}\left[B(u,c^u,c^d,p^M,p;\lambda^{\mathcal{D}})\right] =$$

$$= \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\left(c^u_{it}+c^d_{it}+\sum_{s\in\mathcal{S}}P^s\left[v^s_{it}-\lambda^{\mathcal{D}s}_t p^{M,s}_{it}\right]\right)$$

Initial PCs added to the constraints of DABFC for each (i,t,s)

$$v^s_{it} \geq (2c^q_i\underline{P}_i + c^l_i)p^s_{it} + (c^b_i - c^q_i\underline{P}^2_i)u_{it}$$

$$v^s_{it} \geq (2c^q_i\overline{P}_i + c^l_i)p^s_{it} + (c^b_i - c^q_i\overline{P}^2_i)u_{it}$$

Implementation

- The numerical experiments solved several instances of the DABFC problem with two different procedures:
  - **MIQP** The MIQP solver of Cplex 12.1
  - **PCF** The MILP solver of Cplex 12.1 were the dynamic generation of PCs was implemented by means of the `cutcallback` procedure.
- The same sophisticated tools (valid inequalities, branching rules, . . .) are used for both formulations: MIQP and PCF.
- The tests have been performed on DELL OPTIPLEX GX620 Intel Pentium with 4 CPU and 3.40 GHz, Linux (Suse 11.0)

Test problems

Prob.	$\lvert\mathcal{F}\rvert$	$\lvert\mathcal{S}\rvert$	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{T}\rvert$	# var	# var_{PCF}	# bin	# constr
fcbcuc1	2	2	4	6	264	312	24	428
fcbcuc3	2	2	4	24	1056	1248	96	1688
fcbcuc4	2	4	6	24	2160	2736	144	3970
fcbcuc5	3	4	10	24	3840	4800	240	6596
fcbcuc6	3	5	10	24	4320	5520	240	7796
fcbcuc7	3	10	10	24	6720	9120	240	13796
ismp09	3	61	10	24	31200	45840	240	74996

If we use PCF the problem increases the number of variables in $m = \lvert\mathcal{T}\rvert \cdot \lvert\mathcal{I}\rvert \cdot \lvert\mathcal{S}\rvert$ and the number of constraints in $2 \cdot m$.

CPU times and number of PC

Prob.	MIQP	PCF	PCF/MIQP	# PC
fcbcuc1	0.19	0.14	0.73	166
fcbcuc3	1.31	0.27	0.20	784
fcbcuc4	26.64	1.5	0.05	2271
fcbcuc5	37.27	3.82	0.10	2720
fcbcuc6	21.70	5.47	0.25	3665
fcbcuc7	169,5	33.87	0.20	9687
ismp09	13231.4	1350.89	0.10	45361

1. The problem : Iberian Electricity Market
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   - Case Study characteristics
   - Results
5. Conclusions

Case Study characteristics

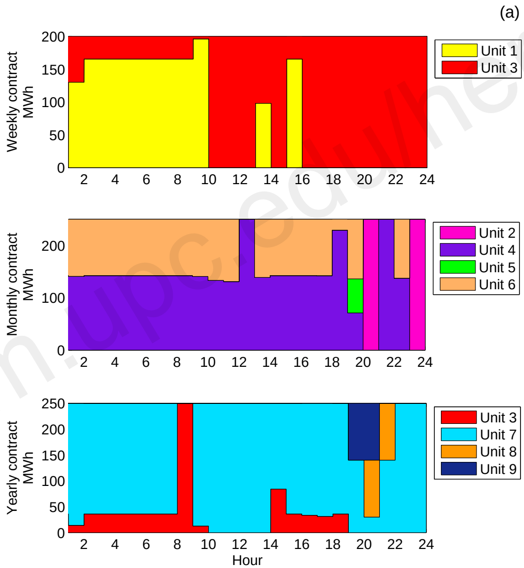
- The DABFC model has been tested using real data from a GenCo and market prices of the MIBEL's DAM.
- 9 thermal generation units (6 coal, 3 fuel) from a Spanish generation company with daily bidding in the MIBEL

$[\overline{P} - \underline{P}]$ (MW)	160-243	250-550	80-260	160-340	30-70
$min_{on/off}$ (h)	3	3	3	4	4

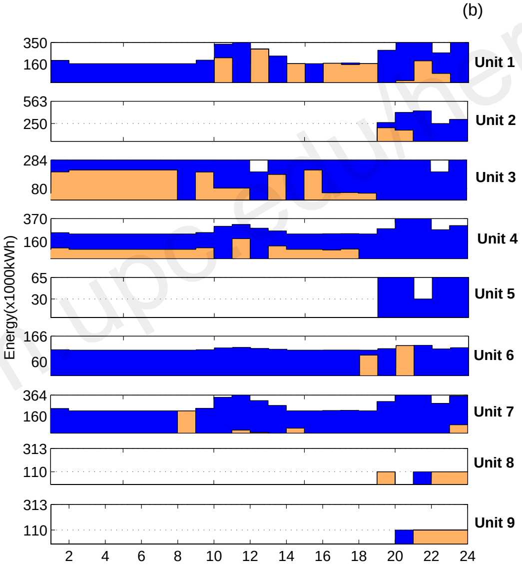
$[\overline{P} - \underline{P}]$ (MW)	60-140	160-340	110-157	110-157
$min_{on/off}$ (h)	3	3	4	4

- Pool of base load BC with 300MWh.
- 3 physical futures contracts with 200MWh, 250MWh and 250MWh.
- Model implemented and solved with AMPL/CPLEX 11.0.

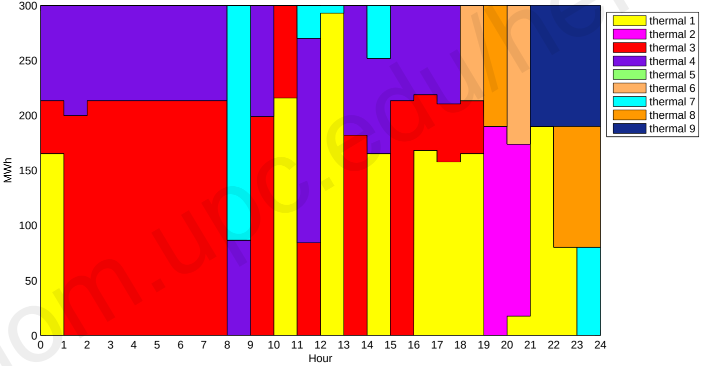
Economic dispatch of each futures contracts f^*_{itj}



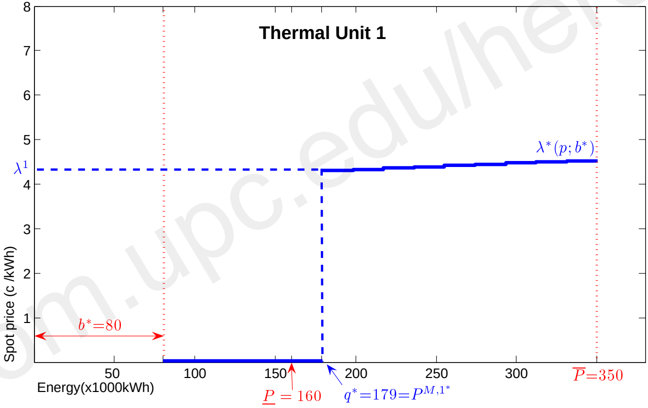
Unit commitment, q^*_{it} and b^*_{it}



Settlement of the bilateral contracts (b^*_{ti})

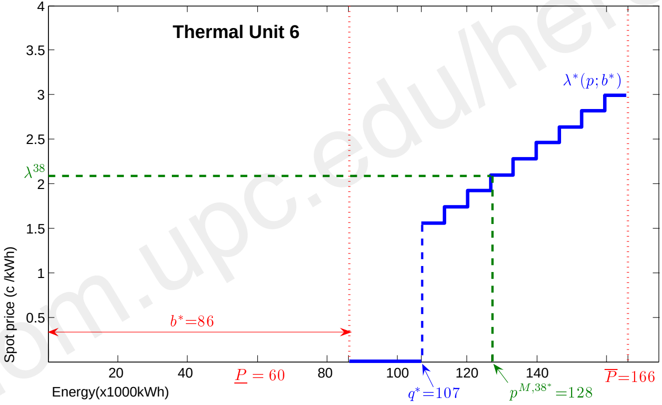


Results: optimal bidding curve



Optimal bidding curve for thermal unit 1 at interval 23

Results: optimal bidding curve



Optimal bidding curve for thermal unit 6 at interval 18

1. The problem : Iberian Electricity Market
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Conclusions (I/III)

- We have presented an Optimal Bidding Model for a price-taker generation company operating both in the MIBEL Derivatives and Day-Ahead Electricity Market.
- The model developed gives the producer:
  - The optimal bid for the spot market.
  - The optimal allocation of the physical futures and bilateral contracts among the thermal units
  - The unit commitment

  following in detail the MIBEL rules.

Conclusions (II/III)

Besides the work presented in this talk, other developments has been/will be undertaken:

- Virtual power plants operation (Heredia, Rider, Corchero 2008).
- Combined cycle units operation (Heredia, Rider, Corchero 2009).
- Hydro generation units operation and Italian electricity market (Vespucci, Corchero, Innorta, Heredia 2009).
- Modeling of the reserve and intraday market (Corchero, Heredia 2010)
- Proximal Bundle Methods (Heredia, Aldasoro, 2010)
- Branch&Fix Coordination (Mijangos, Heredia 2011?)

Conclusions (III/III)

The complete papers can be downloaded from my personal webpage:

http://www-eio.upc.es/ heredia/

and also from the website of the Group on Numerical Optimization and Modeling (UPC):

http://www.gnom.upc.edu/

Electricity Market Optimization: finding the best bid through stochastic programming.

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