# Perspective cuts for solving the optimal electricity market bid problem with bilateral contracts



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# Outline

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# Introduction



- In liberalized electricity markets, a Generation Company (GenCo) must build an hourly bid that is sent to the market operator, who selects the lowest price among the bidding companies in order to match the pool load.
- GenCos need to know the prices at which the energy will be paid in order to decide how to bid and how to schedule their resources for maximizing their profit.

# Introduction

- But, the market price is a random variable whose realization is only known once the market has been cleared.
- A forecast procedure based on factor models gives us the probability distribution of this random variable (Corchero et al., 2010).
- The set of scenarios built from the forecasting results are used to feed a stochastic optimization model that finds the optimal day-ahead bid of a price-taker GenCo operating in the MIBEL and holding bilateral and future contracts.

# Introduction

- The optimization model used in this work extends the model of Heredia et al. (2010), with the addition of physical future contracts.
- This model is a Mixed-Integer Quadratic Program (MIQP), which is difficult to solve efficiently, especially for large-scale instances.
- We approximate the quadratic objective function by means of perspective cuts (Frangioni and Gentile, 2006), so that this problem can be solved as a Mixed-Integer Linear Program (MILP) by general-purpose MILP solvers.

# Parameters

Day-Ahead electricity market with Bilateral and Futures Contracts (DABFC) model is built for a price-taker GenCo owning a set of thermal generation units $\mathcal{I}$ that bid to the $t \in \mathcal{T} = \{1, 2, .., 24\}$ hourly auctions of the DAM.

## Parameters for the $i^{th}$ thermal unit

- generation costs with constant, linear and quadratic coefficients, $c_i^b$ (€), $c_i^l$ (€/MWh) and $c_i^q$ (€/MWh$^2$).
- $\overline{P}_i$ and $\underline{P}_i$ are the upper and lower bound on the energy generation (MWh).
- start-up, $c_i^{on}$, and shut-down, $c_i^{off}$, costs (€).
- minimum operation and minimum idle time, $t_i^{on}$ and $t_i^{off}$.

# Parameters

## Base load futures contract $j \in \mathcal{F}$

- $L_j^{FC}$: amount of energy (MWh) to be procured each interval of the delivery period by the set $U_j$ of generation units.
- $\lambda_j^{FC}$: price of the contract (€/MWh).

## Bilateral contract $k$

- $L_{kt}^{BC}$: amount of energy (MWh) to be procured at interval $t$ of the delivery period by the whole set of generation units.
- $\lambda_k^{BC}$: price of the contract (€/MWh).

# Parameters



The energies $L_j^{FC}$ and $L_{kt}^{BC}$ should be integrated in the MIBEL's day-ahead bid observing the two following rules:

1. If generator $i$ contributes with $f_{itj}$ MWh at period $t$ to the coverage of the FC $j$, then the energy $f_{itj}$ must be offered to the pool for free (instrumental price bid).
2. If generator $i$ contributes with $b_{it}$ MWh at period $t$ to the coverage of the BCs, then the energy $b_{it}$ must be excluded from the bid to the day-ahead market. Unit $i$ can offer its remaining production capacity $\overline{P}_i - b_{it}$ to the pool.

# Variables



## $1^{st}$ stage variables for $t \in \mathcal{T}$ and $i \in \mathcal{I}$

- The unit commitment variables: $u_i^t$ (binary) , $c_{it}^u$, $c_{it}^d$
- The instrumental price offer bid variables: $q_{it}$.
- The scheduled energy for futures contract $j \in \mathcal{F}$ variables: $f_{itj}$.
- The scheduled energy for bilaterals contract variables: $b_{it}$.

## $2^{nd}$ stage variables for each scenario $s \in \mathcal{S}$

- Total generation: $p_{it}^s$
- Matched energy in the day-ahead market: $p_{it}^{\mathcal{M},s}$

# Bilateral and future contracts constraints

## Future contract constraints for $j \in \mathcal{F}$ , $t \in \mathcal{T}$

$$\sum_{i \in U_{jt}} f_{itj} = L_j^{FC}$$

$$f_{itj} \geq 0 \,, \quad i \in \mathcal{I}$$

## Bilateral contract constraints for $t \in \mathcal{T}$

$$\sum_{i \in U_t} b_{it} = L_t^{BC}$$

$$0 \leq b_{it} \leq \overline{P}_i \,, \quad i \in \mathcal{I}$$

**Matched energy for $i \in \mathcal{I}$ , $t \in \mathcal{T}$ , $s \in \mathcal{S}$**

$$p_{it}^{\mathcal{M},s} \leq \overline{P}_i u_{it} - b_{it}$$

$$p_{it}^{\mathcal{M},s} \geq q_{it}$$

**Instrumental price bid for $i \in \mathcal{I}$ , $t \in \mathcal{T}$**

$$q_{it} \geq \underline{P}_i u_{it} - b_{it}$$

$$q_{it} \geq \sum_{j \,|\, i \in U_j} f_{itj}$$

$$q_{it} \geq 0$$

**Total generation constraints for $t \in \mathcal{T}$ , $i \in \mathcal{I}$ , $s \in \mathcal{S}$**

$$p_{it}^{s} = b_{it} + p_{it}^{\mathcal{M},s}$$

# Unit commitment constraints

This formulation follows that proposed by Carrion (2006).

**Start-up and shut-down costs for $i \in \mathcal{I}$**

$$c^u_{it} \geq c^{on}_i [u_{it} - u_{i,(t-1)}] \; , \; t \in \mathcal{T} \setminus \{1\}$$
$$c^d_{it} \geq c^{off}_i [u_{i,(t-1)} - u_{it}] \; , \; t \in \mathcal{T} \setminus \{1\}$$
$$c^u_{it}, c^d_{it} \geq 0 \; , \; t \in \mathcal{T}$$

**Parameters $G_i$ and $H_i$: on- and off-initial time periods for $i \in \mathcal{I}$**

$$\sum_{j=1}^{G_i} (1 - u_{ij}) = 0 \quad \text{and} \quad \sum_{j=1}^{H_i} u_{ij} = 0$$

# Unit commitment constraints

## $t^{on}$ and $t^{off}$: minimum up and down time for $i \in \mathcal{I}$

$$\sum_{n=t}^{t+t_i^{on}-1} u_{in} \geq t_i^{on}[u_{it} - u_{i,(t-1)}] , \quad t = G_i + 1, \ldots, |\mathcal{T}| - t_i^{on} + 1$$

$$\sum_{n=t}^{t+t_i^{off}-1} (1 - u_{in}) \geq t_i^{off}[u_{i,(t-1)} - u_{it}] , t = H_i + 1, \ldots, |\mathcal{T}| - t_i^{off} + 1$$

# Unit commitment constraints

**For the last $t^{on} - 1$ and $t^{off} - 1$ time periods and $i \in \mathcal{I}$**

$$\sum_{n=t}^{|\mathcal{T}|} (u_{in} - [u_{it} - u_{i(t-1)}]) \geq 0 \; , \; t = |\mathcal{T}| - t_i^{on} + 2, \ldots, |\mathcal{T}|$$

$$\sum_{n=t}^{|\mathcal{T}|} (1 - u_{in} - [u_{i(t-1)} - u_{it}]) \geq 0 \; , \; t = |\mathcal{T}| - t_i^{off} + 2, \ldots, |\mathcal{T}|$$

# Objective function

$$\min_{p,q,f,b} \mathrm{E}_{\lambda^{\mathcal{D}}}\left[B(u, c^u, c^d, p^{\mathcal{M}}, p; \lambda^{\mathcal{D}})\right] = \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\Bigg(c^u_{it} + c^d_{it} + c^b_{it}u_{it} +$$

$$+ \sum_{s\in\mathcal{S}} P^s\left[(c^l_i p^s_{it} + c^q_i (p^s_{it})^2) - \lambda^{\mathcal{D}s}_t p^{\mathcal{M},s}_{it}\right]\Bigg)$$

- Model DABFC is the deterministic equivalent program associated with the two-stage stochastic problem with a set $\mathcal{S}$ of scenarios for the spot price $\lambda^{\mathcal{D}}_t$, where $t \in \mathcal{T}$.
- This deterministic program is a convex MIQP with a well defined global optimal solution.

# Motivation



- In order to solve DABFC by commercial MILP software, the quadratic part of the objective function must be linearized.
- Since the sum of the probabilities $P^s$ is one, we can include the products $c^b_{it} u_{it}$ in the quadratic parenthesis for each block $(i, t, s)$ in this way:

$$c^l_i p^s_{it} + c^q_i (p^s_{it})^2 + c^b_{it} u_{it},$$

  where the variables $u_{it}$ are binary. For notational simplicity in this paragraph we drop the indices.
- The issue is then how to best represent the quadratic function

$$f(p, u) = c^q p^2 + c^l p + c^b u$$

  by means of a piecewise-linear one.

# Motivation

- There is an effective way based on ideas developed by Frangioni and Gentile (2006).
- The function $f(p, u)$ is only relevant at points $(p, u)$ of its (disconnected) domain $\mathcal{D} = [0, 0] \cup \left[ [\underline{P}, \overline{P}] \times \{1\} \right]$.
- However, standard branch-and-cut approaches typically solve the continuous relaxation of the mixed problem, where $u \in [0, 1]$ instead of $\{0, 1\}$, in order to obtain lower bounds on the optimal value.
- This makes sense to use the convex envelope of $f(p, u)$ over $\mathcal{D}$, that is, the convex function with the smallest (in set-inclusion sense) epigraph containing that of $f(p, u)$.

# Convex envelope

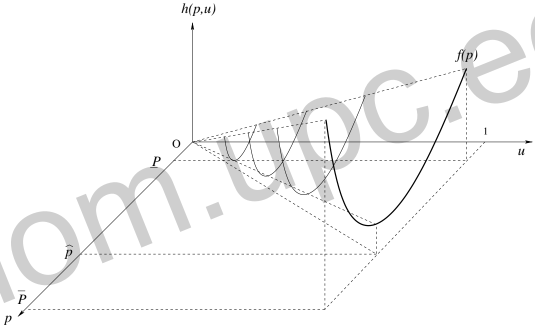


- As is showed by Frangioni and Gentile (2006) the convex envelope is

$$h(p,u) = \begin{cases} 0, & \text{if } (p,u) = (0,0) \\ \dfrac{c^q p^2}{u} + c^l p + c^b u, & \left\{ \begin{array}{c} \text{if } u\underline{P} \leq p \leq u\overline{P}, \\ \text{for } u \in (0,1] \end{array} \right\} \\ +\infty, & \text{otherwise.} \end{cases}$$

# Convex envelope






This function is strongly related with the perspective-function $\breve{f}(p,u) = uf(p/u)$ of $f(p) = c^q p^2 + c^l p + c^b$, which is convex if $f(p)$ is convex.

# Definition

- $h(p,u) \geq f(p,u)$ for $0 < u \leq 1$, i.e. $h$ is a tighter objective function than $f$ for the continuous relaxation.
- As is well-known, every convex function is the point-wise supremum of affine functions.
- In fact, the epigraph of $h$ is composed of all and only triples $(v,p,u)$ satisfying $u\underline{P} \leq p \leq u\overline{P}$, $0 \leq u \leq 1$ and the infinite system of linear inequalities

$$v \geq (2c^q\widehat{p} + c^l)p + (c^b - c^q\widehat{p}^2)u$$

  taking $\widehat{p} \in [\underline{P}, \overline{P}]$.
- For each $\widehat{p}$ we have a inequality so-called a perspective cut (PC), which is the unique supporting hyperplane to the function passing by $(0,0)$ and $(\widehat{p},1)$.

# PC formulation (PCF)

- PC formulation (PCF) lies in choosing these supporting hyperplanes and using as an objective function the polyhedral function that is the point-wise maximum of the corresponding linear functions.
- An small set of initial PCs is chosen to solve the problem with the continuous relaxation.
- When $u^* > 0$, check whether the solution $(v^*, p^*, u^*)$ satisfies the PC for $\widehat{p} = p^*/u^*$; if not, the obtained cut can be added to PCF.
- PCF starts with only two pieces, the ones corresponding with $\underline{P}$ and $\overline{P}$; additional cuts are then dynamically generated when needed as described in the previous paragraph.

# PC formulation (PCF)

## Objective function for PCF

$$\min_{p,q,f,b} \mathrm{E}_{\lambda^{\mathcal{D}}} \left[ B(u, c^u, c^d, p^{\mathcal{M}}, p; \lambda^{\mathcal{D}}) \right] =$$

$$= \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \left( c^u_{it} + c^d_{it} + \sum_{s \in \mathcal{S}} P^s \left[ v^s_{it} - \lambda^{\mathcal{D}s}_t p^{\mathcal{M},s}_{it} \right] \right)$$

## Initial PCs added to the constraints of DABFC for each $(i, t, s)$

$$v^s_{it} \geq (2c^q_i \underline{P}_i + c^l_i) p^s_{it} + (c^b_i - c^q_i \underline{P}^2_i) u_{it}$$

$$v^s_{it} \geq (2c^q_i \overline{P}_i + c^l_i) p^s_{it} + (c^b_i - c^q_i \overline{P}^2_i) u_{it}$$

# Implementation

- In our numerical tests we have used Cplex 12.1, which allows one to directly input the DABFC problem as a Mixed-Integer Linearly Constrained Quadratic Program and solve it as a MIQP. Moreover, for PCF the dynamic generation of PCs can be easily implemented by means of the `cutcallback` procedure.
- Thus, apart from the basic formulation, the same sophisticated tools (valid inequalities, branching rules, . . . ) are used for both formulations: MIQP and PCF.

# Implementation



- A few differences remain: e.g. the need for invoking the callback functions disables the — allegedly — more efficient dynamic search of Cplex 12.1 for PC, whereas it is used when the DABFC problem is solved by Cplex as a MIQP.
- Apart from these, the very same machinery is used with both formulations, allowing a fair comparison.
- The tests have been performed on DELL OPTIPLEX GX620 Intel Pentium with 4 CPU and 3.40 GHz, Linux (Suse 11.0)

# Test problems

| Prob. | $\|\mathcal{F}\|$ | $\|\mathcal{S}\|$ | $\|\mathcal{I}\|$ | $\|\mathcal{T}\|$ | # var | # $\text{var}_{PCF}$ | # bin | # constr |
|---|---|---|---|---|---|---|---|---|
| fcbcuc1 | 2 | 2 | 4 | 6 | 264 | 312 | 24 | 428 |
| fcbcuc3 | 2 | 2 | 4 | 24 | 1056 | 1248 | 96 | 1688 |
| fcbcuc4 | 2 | 4 | 6 | 24 | 2160 | 2736 | 144 | 3970 |
| fcbcuc5 | 3 | 4 | 10 | 24 | 3840 | 4800 | 240 | 6596 |
| fcbcuc6 | 3 | 5 | 10 | 24 | 4320 | 5520 | 240 | 7796 |
| fcbcuc7 | 3 | 10 | 10 | 24 | 6720 | 9120 | 240 | 13796 |
| ismp09 | 3 | 61 | 10 | 24 | 31200 | 45840 | 240 | 74996 |

(Note: # Bilateral contracts = 2, except for ismp09, in this case is 3.)

# CPU times and number of PC

| Prob. | MIQP | PCF-MILP | # PC |
|---|---|---|---|
| fcbcuc1 | 0.19 | 0.14 | 166 |
| fcbcuc3 | 1.31 | 0.27 | 784 |
| fcbcuc4 | 26.64 | 1.5 | 2271 |
| fcbcuc5 | 37.27 | 3.82 | 2720 |
| fcbcuc6 | 21.70 | 5.47 | 3665 |
| fcbcuc7 | 169,5 | 33.87 | 9687 |
| ismp09 | 13231.4 | 1350.89 | 45361 |

# Summary

- In each node of the Branch-and-Cut the quadratic part of the objective function have been replaced by the convex envelope of its epigraph, which is likewise fixed by the polyhedral function that is the point-wise maximum of the linear functions of the perspective cuts.
- If we use PCF the problem increases the number of variables in $m = |\mathcal{T}| * |\mathcal{I}| * |\mathcal{S}|$ and the number of constraints in $2m$.
- Perspective cut formulation is significantly efficient in the solution of DABFC problems.