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# A multistage formulation for generation companies in a multi-auction electricity market

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In this paper, we deal with the definition of a decision model for a producer operating in a multi-auction electricity market. The decisions to be taken concern the commitment of the generation plants and the quantity of energy required to offer to each auction and to cover the bilateral contracts. We propose a multistage stochastic programming model in which the randomness of the clearing prices is represented by means of a scenario tree. The risk is modelled using a Conditional Value at Risk term in the objective function. Experimental results are reported to show the validity of our model and to discuss the influence of the risk parameters on the optimal value.

*Keywords*: electricity market; multistage stochastic programming; bidding strategy; Conditional Value at Risk.

## 1. Introduction

In the last few years, the liberalization process that has spread over many electricity markets around the world has generated deep changes in an economic context that has been very conservative for a long time. The electricity industry is evolving into a distributed and competitive framework in which market forces drive the price of electricity both on the buying and the selling sides. The main difference with respect to the previous structure has been the introduction of competition into the different phases that characterize the electricity system.

In this new context, the operators have to face new operational problems for the efficient management of their activities since new issues, such as the market price forecasting and the risk management, have become critical (Arroyo & Conejo, 2000). Power generation planning and operation that explicitly include both randomness and dynamics of electricity markets into a mathematical model is already a consolidated approach in the scientific literature. Earlier works have focused on the use of the two-stage stochastic framework to incorporate the randomness into the mathematical models of well-known

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problems such as the unit commitment, the capacity expansion and the bidding strategy definition (Malcolm & Zenios, 1994; Beraldi *et al.*, 2007; Nowak *et al.*, 2005). However, the two-stage framework succeeds to capture efficiently the stochastic aspect but only partially the dynamics of the problem because of the repetitive nature of the electricity market. A first improvement has been done by developing two-stage multi-period models, but afterwards, the scientific community has realized that the multistage framework is becoming the most appropriate framework to capture both the dynamic and the stochastic aspects of power planning problems. These issues are critical also in the financial field for which much more decision approaches based on the multistage stochastic programming framework have been proposed (see Mulvey & Ziemba, 1998; Consigli & Dempster, 1998; Kouwenberg & Zenios, 2001).

Several contributions concerning the definition of effective decision tools for the market operators have been already proposed. Among these, we cite the work of Fleten *et al.* (2002) that proposes a model that combines hydroelectric system generation as well as investments in financial markets. Hochreiter *et al.* (2006) have proposed a solution for the problem of big consumer portfolio definition. The problem of structuring portfolios of bilateral energy trading contracts has been discussed in Pizarro Romero *et al.* (2007) and a multistage stochastic mixed 0/1 model for its solution has been presented. In Flach (2007), a methodology for the strategic bidding problem in the case of a price-maker hydro agent with multiple plants based on a multistage programming approach has been proposed. The problem of scheduling of wind power generation in a detailed representation of the grid operation has been discussed in Barth (2007) and the problem has been formulated as a three-stage stochastic problem.

Multistage mixed-integer models for the power scheduling in hydrothermal systems have been proposed in Gröwe-Kuska *et al.* (2002) and Nowak & Römisch (2000) and solution methods based on Lagrangian relaxation have been developed. Earlier, Takriti *et al.* (1996) have solved the unit commitment problem by means of a multistage stochastic model when load demand is uncertain.

In such a complex context, most of the contributions consider the possibility of trading in a single electricity auction ignoring the opportunity of operating in multi-auction markets (a detailed description of the multi-auction structure is reported in Section 2). Little attention, indeed, has been paid to the multistage stochastic power scheduling in a multi-auction and/or multi-market environment. In this context, we are aware of two different contributions. The first one, due to Plazas *et al.* (2005), defines a bidding strategy for a producer participating in a sequence of three spot markets. The state of the generation units is considered to be known in advance except for the units dedicated to the automatic generation control market. The resulting multistage model is made computationally attractable through a scenario reduction approach and solved by using general purpose software package. The second contribution, due to Triki *et al.* (2005), proposes a capacity allocation approach by means of which the GENeration COmpany (GENCO) can decide not only the quantities to offer to each of the available auctions but also the commitment of each unit generation. Their mathematical model results to be a multistage stochastic non-linear program and a general purpose solver has been used for its solution as well.

This paper can be considered as an extension to the contribution of this last reference. However, while the formulation proposed in Triki *et al.* (2005) deals just with the capacity allocation problem, this work broadens the interest to the definition of a bidding strategy. The new features aim at proposing a more realistic and general representation of the decision process allowing the GENCO to maximize its profits and, at the same time, to monitor the risk due to the market operations. More specifically, with respect to Triki *et al.* (2005), the following critical issues have been added to the decision process:

- ensure the respect of previously committed bilateral contracts in the energy balance;
- introduce a more accurate representation of the production units' dynamics and costs;

- define a units' generation schedule for the auctions offers and bilateral contracts energy requirements by means of specific selling bids on the day-ahead market (DAM);
- include the possibility of buying energy on the adjustment market (AM) in order to 'correct' undesirable (DAM) outcomes;
- consider and model explicitly the zonal market paradigm;
- incorporate a risk aversion tool of the GENCO by means of a modern risk measure like the Conditional Value at Risk (CVaR).

The remaining part of the paper is organized as follows. Section 2 contains an overview of the main issues of multi-auction electricity markets. Section 3 reports a more detailed description of the decision problem. In particular, a multistage mathematical model is presented and some critical issues, like the risk management, are discussed. Section 4 reports the computational experiments that we have carried out in order to validate the effectiveness of the proposed decision approach. Some concluding remarks end the paper.

## 2. Market structure

In many countries, e.g. Italy, the new competitive paradigm provides two ways for the GENCOs to sell electric energy: (i) bilateral contracts that are independent agreements between producers and eligible consumers and (ii) power pool, i.e. an e-commerce marketplace organized in several consecutive sessions where producers and consumers submit production and consumption bids, respectively. More specifically, the power pool may be organized in three different sessions, each with its peculiar auction mechanism:

- DAM for the wholesale trading of energy between producers (GENCOs) and wholesale customers. This market usually takes place in the morning of the day ahead of the delivery day;
- AM, where market participants may revise the schedules resulting from the DAM, by submitting additional demand bids or supply offers. This market takes place immediately after the DAM, usually in the afternoon;
- Ancillary services market (ASM), where market participants submit offers/bids to increase or decrease injection or withdrawal for each elementary time period. The market grid operator uses these offers to correct the schedules that violate the transmission limits on the grid and to create reserve margins for the following day or to balance in real time the system in case of deviations from the schedules. Based on the jurisdiction to be considered, these services may be organized in only one or in different sessions.

Moreover, it is worthwhile noting that after each submission deadline, the market operator activates the market clearing process (see Fig. 1). For each hour of the operating day, those offers/bids that maximize the value of the transactions (economic merit criterion) will be accepted provided that the transmission limits between the zones are not violated (see, e.g. Beraldi *et al.*, 2004).

In other terms, the market operates with a zonal model, which has been successfully tested in many European countries as well as in almost all the liberalized markets of the USA and Oceania. If at least one transmission limit is violated, then the algorithm will 'split' the market into two market zones: one exporting zone, including all the zones lying above the constraint, and one importing zone, including all the zones lying below the constraint. This splitting process is repeated in each zone, building a supply curve for each market zone (including all supply offers submitted in the same zone as well as the

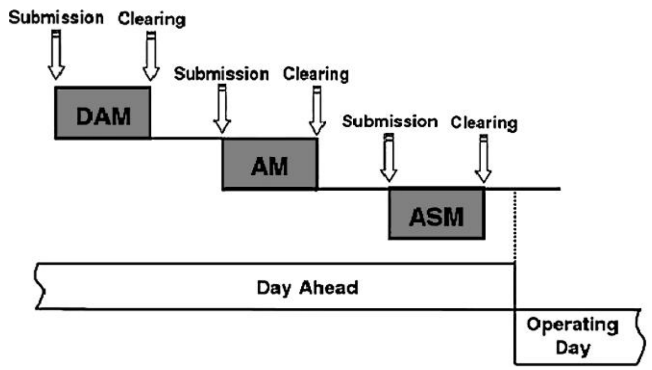


FIG. 1. Sequence of market sessions.

maximum imported quantity) and a demand curve (including all demand bids submitted in the same zone as well as the maximum exported quantity). The result will be a zonal clearing price, $P_z$, different in each zone $z$, at which all the supply offers referred to a zone will be valued. In particular, $P_z$ will be higher in importing zones and lower in exporting ones. If, as a result of this solution, additional constraints are violated, then the market splitting process will be replicated within the already created zones, until the market result satisfies the grid constraints. This paradigm reflects a hybrid model of market structure that offers a true customer choice and encourages the creation of a wide variety of services and price options to best meet individual customer needs. An overview of the liberalized electric market features is reported in Sheblé (1999), whereas a detailed description of the Italian market structure can be found in Gestore del mercato elettrico (2002).

## 3. Problem formulation

The problem we are facing is related to the definition of the optimal quantities to be bought/sold by a GENCO operating in a multi-auction market like the one described in Section 2. A model has been thus proposed as a tool helpful for a GENCO to commit its units of production and to decide how much capacity each unit must dedicate in each market session and for the fulfillment of bilateral contracts. As far as to the bidding process, selling offers are formulated taking into account the different time horizons of the market sessions, i.e. formulating first offers for DAM, waiting for its clearing, then bidding for AM, being known the acceptance of the previous offers, waiting again for the corresponding AM clearing and then formulating the last offer in the ASM, allocating all the residual capacity from the acceptance of the other markets. The different time horizons of the bidding processes for the three market sessions have been modelled by means of constraints simulating the subsequent decisions the GENCO must make, linking each time-dependent decision with previous and successive ones, by means of an intuitive scenario tree structure. This scenario tree models clearing process, showing at each stage a number of nodes representing all the likely alternatives of acceptance, each node containing information about clearing price of the session under observation and the corresponding acceptance percentage of the relating offer. We have therefore assumed that the seller is a price-taker, i.e. with no possibility to exercise market power to affect the clearing price. This assumption is realistic for small sellers and for markets characterized with no collusions and entry barriers. Moreover, we have not considered the definition of a supply curve, i.e. a function with (complex) combinations of prices and quantities (as discussed in Anderson & Philpott, 2002, and also in Fleten & Kristoffersen , 2007, and in the references therein), but rather we simply assume that the GENCO offers a quantity of energy at a given price level. The offered price is either the units' output marginal cost or simply a user-defined price that ensures a reasonable bid acceptance confidence level.

Moreover, a further observation must be made as far as ASM. Although this market session closes on the day ahead of the operating day, the process of acceptance of offers/bids takes place in two phases: (i) immediately after the close of the sitting, i.e. as planned, when the accepted offers/bids are used to revise the injection and withdrawal schedules resulting from the DAM and the AM, so as to relieve any residual congestion not managed in such markets and to create the reserve margins needed to guarantee the security of the system and (ii) throughout the day of delivery (i.e. in real time) when offers/bids are accepted in order to balance the system in real time. We have decided to cumulate these two different phases since we do not simulate a real-time operation but a day-ahead planning. Nonetheless, it would be straightforward to involve additional variables with corresponding constraints and no further complexities would be involved.

A GENCO that operates in such a complex market with the aim of maximizing its profits should determine the optimal bidding strategy and capacity allocation both at single-unit level and at corporate level, taking into account several issues; in addition to the restrictions imposed by the units' physical/operational limitations and market regulations, a GENCO may want to include strategic objectives such as diversification and market niche. The complexity of this decision process increases with both the number of units and the number of market session, but the real critical issue is the explicit representation of the uncertainty. Some of the most important data are inherently uncertain: at the beginning of every auction, the clearing price and quantities bought/sold are not known.

The aim of this work is the definition of a mathematical model to support the decision process of a GENCO that wants to allocate its production capacity in the most profitable way but respecting a certain level of risk aversion. As introduced above, this decision process is dynamic since there are many decision phases corresponding to the multi-auction framework, as depicted in Fig. 1.

Moreover, the decision process is also made under uncertainty since the amounts of energy effectively sold, and the clearing prices, depend on the market auction outcomes. An effective decision approach cannot ignore this uncertainty but, on the contrary, should take into account all the random events that may occur.

These two main characteristics have suggested the adoption of multistage stochastic programming as a modelling framework for this problem. In order to represent the uncertainty, i.e. the possible realizations of clearing prices, we have adopted a scenario tree representation (see Fig. 2). The root node stands for the first stage and corresponds to the immediately observable deterministic data. The nodes in successive stages correspond to possible outcomes of the various market sessions and each one is characterized by a certain probability of occurrence. Moreover, each node $k$, except the root node, has a unique predecessor $p(k)$ in the preceding stage and a finite number of successors in the next stage. Nodes without successors are the 'leaves' of the tree and are as many as the number of scenarios: a scenario can be seen, thus, as a path from the root node to a leaf and represents a joint outcomes of the problem uncertain data over all the market sessions.

The scenario tree is also associated with the sequential decision process, so that to each node corresponds a decision variable that depends not only on the previous decisions but also on the outcomes of the random data so far observed as well.

### 3.1 *Problem data*

In this section, we briefly introduce the problem parameters starting from the (deterministic) generators characteristics and ending with the (uncertain) market outcomes.

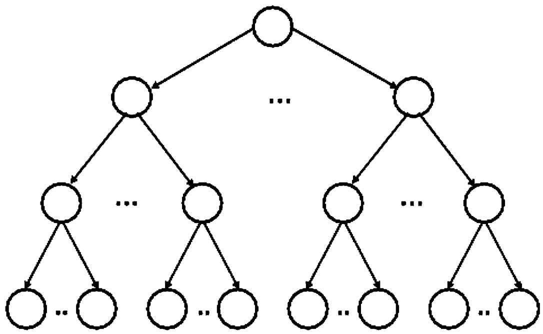


FIG. 2. Scenario tree representation.

- $T$ planning horizon, usually 1 day divided into 24 elementary hourly period;
- $I$ set of available production units, each one located into a particular market zone;
- $Q_i^{\min}$, $Q_i^{\max}$ minimum and maximum capacity of thermal unit $i$ (MWh);
- $\alpha_i$, $\beta_i$ fixed and linear component of the variable cost of unit $i$ (Euro and Euro/MWh, respectively);
- $\mathrm{SU}_i$, $\mathrm{SD}_i$ fixed start-up and shutdown costs of unit $i$ (Euro);
- $\mathrm{UT}_i$, $\mathrm{DT}_i$ minimum uptime and downtime of unit $i$ (h);
- $\mathrm{Up}_{i0}$, $\mathrm{Down}_{i0}$ number of time periods unit $i$ has been online or offline at the beginning of the planning horizon (hours);
- $U_{i0}$ initial status of unit $i$ (1 if it is online, 0 otherwise);
- $Q_t^{\mathrm{bil}}$ total quantity of energy committed for bilateral contracts for period $t$ (MWh);
- $R_t^{\mathrm{bil}}$ revenues from bilateral contracts for time period $t$(Euro);
- $S$ set of likely outcomes (‘scenarios’) of the DAM;
- $\eta^s$ occurrence probability of scenario $s \in S$;
- $\lambda_{it}^s$ DAM zonal price at period $t$ under scenario $s$ for unit $i$ (depending on the zone unit $i$ belongs to) (Euro/MWh);
- $\gamma_{it}^s$ percentage of bid acceptance in the DAM for unit $i$ in period $t$ under scenario $s$;
- $L$ set of likely outcomes (‘scenarios’) of the AM;
- $\pi^l$ occurrence probability of scenario $l \in L$;
- $\mu_{it}^l$ AM zonal price at period $t$ under scenario $l$ for unit $i$ (depending on the zone unit $i$ belongs to) (Euro/MWh);
- $\nu_t^l$ AM clearing price at period $t$ under scenario $l$ (unique for all the maket zones) (Euro/MWh);
- $\delta_{it}^{l+}$ percentage of selling bid acceptance in the AM for unit $i$ in period $t$ under scenario $l$;
- $\delta_{it}^{l-}$ percentage of purchasing bid acceptance in the AM for unit $i$ in period $t$ under scenario $l$;

- $V$ set of likely outcomes (‘scenarios’) of the ASM;
- $\theta^{\upsilon}$ occurrence probability of scenario $\upsilon \in V$;
- $\varsigma_t^{\upsilon}$ ASM clearing price at period $t$ under scenario $\upsilon$ (Euro/MWh);
- $\rho_{it}^{\upsilon}$ percentage of bid acceptance in the ASM for unit $i$ in period $t$ under scenario $\upsilon$.

### 3.2 *Decision variables*

The model variables represent the decisions that the GENCO has to take in order to plan its bidding strategy and, according with the market outcomes, the production schedule. More specifically, the GENCO has to define first which units to commit for each time period and the quantities to offer in each market sessions. We indicate with $x_{it}$ the quantity of output of unit $i$ to offer in the DAM for time period $t$. Moreover, the production decisions must take into account the presence of bilateral contracts stipulated time before the planning horizon. To satisfy this demand, the producers can define for each time period one or more offers on the DAM at price 0. The quantity of energy of these offers are decision variables in our approach ($x_{it}^{\text{bil}}$). This issue introduces another decision level for the GENCO who has to decide which unit (or units) each contract has to be referred to. In terms of multistage notation, all the variables $x_{it}$ and $x_{it}^{\text{bil}}$ are first-stage decisions (Stage 0 in the scenario tree in Fig. 2).

On the basis of each DAM outcome $s$ (i.e. for each child of the root node in Fig. 2), the GENCO could correct its offers, with both selling and buying bids. We indicate with $y_{it}^{s+}$ the quantity of energy of the selling bid on the AM for the unit $i$ for time period $t$ under scenario $s$. Similarly, $y_{it}^{s-}$ represents the buying bid on the AM related to unit $i$ for time period $t$ under scenario $s$. Both $y_{it}^{s+}$ and $y_{it}^{s-}$ can be considered second-stage variables. According to the possible outcomes $l$ of the AM, the GENCO can offer for each unit $i$ and for each time period $t$ a quantity of energy on the ASM $z_{it}^{l}$ (third-stage variables). Finally, according to the observed outcome $\upsilon$ of the ASM, the energy quantity to be produced by each unit for each time period ($Q_{it}^{\upsilon}$) is defined (fourth-stage variables).
We choose to consider also as first-stage decisions the state variables $U_{it}$ according to a realistic view of the operational profile definition for the generators. In order to represent the start-up and the shutdown states of each unit $i$ from time period $t-1$ to time period $t$, we introduce the binary variables $\Delta_{it}^{+}$ and $\Delta_{it}^{-}$, which are strictly linked to the state variables $U_{it}$, as it will be explained later. Thus, our model's decision variables can be summarized as follows:

- $x_{it}$ production of unit $i$ to offer in the DAM for time period $t$;
- $x_{it}^{\text{bil}}$ production of unit $i$ to offer in the DAM for time period $t$ for bilateral contracts (bids at price 0);
- $U_{it}$ (binary) state variable for unit $i$ for time period $t$;
- $\Delta_{it}^{+}$ (binary) start-up variable for unit $i$ for time period $t$;
- $\Delta_{it}^{-}$ (binary) shutdown variable for unit $i$ for time period $t$;
- $y_{it}^{s+}$ quantity of energy of the selling bid on the AM for unit $i$ for time period $t$ under scenario $s$;
- $y_{it}^{s-}$ quantity of energy of the buying bid on the AM for unit $i$ for time period $t$ under scenario $s$;
- $z_{it}^{l}$ quantity of energy of the selling bid on the ASM for unit $i$ for time period $t$ under scenario $l$;
- $Q_{it}^{\upsilon}$ production of unit $i$ for time period $t$ under scenario $\upsilon$.

### 3.3 *Model constraints*

Most of the model constraints are similar to those proposed in the work of Triki *et al.* (2005), but they take into account a larger set of operational restrictions and the presence of new decision variables:

$$x_{it} + x_{it}^{\text{bil}} \leqslant Q_i^{\max} U_{it} \qquad \forall i, \forall t, \qquad (3.1)$$

$$\sum_{i=1}^{I} x_{it}^{\text{bil}} = Q_t^{\text{bil}} \qquad \forall t, \qquad (3.2)$$

$$y_{it}^{s+} \leqslant Q_i^{\max} U_{it} - x_{it}^{\text{bil}} - \gamma_{it}^{s} x_{it} \qquad \forall i, \forall t, \forall s, \qquad (3.3)$$

$$y_{it}^{s-} \leqslant \gamma_{it}^{s} x_{it} \qquad \forall i, \forall t, \forall s, \qquad (3.4)$$

$$y_{it}^{s+} \leqslant M \varphi_{it}^{s+} \qquad \forall i, \forall t, \forall s, \qquad (3.5)$$

$$y_{it}^{s-} \leqslant M \varphi_{it}^{s-} \qquad \forall i, \forall t, \forall s, \qquad (3.6)$$

$$\varphi_{it}^{s+} + \varphi_{it}^{s-} \leqslant 1 \qquad \forall i, \forall t, \forall s, \qquad (3.7)$$

$$z_{it}^{l} \leqslant Q_i^{\max} U_{it} - x_{it}^{\text{bil}} - \gamma_{it}^{p(l)} x_{it} - \delta_{it}^{l+} y_{it}^{p(l)+} + \delta_{it}^{l-} y_{it}^{p(l)-} \qquad \forall i, \forall t, \forall l, \qquad (3.8)$$

$$Q_{it}^{\upsilon} = x_{it}^{\text{bil}} + \gamma_{it}^{p(p(\upsilon))} x_{it} + \delta_{it}^{p(\upsilon)+} y_{it}^{p(p(\upsilon))+} - \delta_{it}^{p(\upsilon)-} y_{it}^{p(p(\upsilon))-} + \rho_{it}^{\upsilon} z_{it}^{p(\upsilon)} \qquad \forall i, \forall t, \forall \upsilon, \qquad (3.9)$$

$$Q_{it}^{\upsilon} \geqslant Q_i^{\min} U_{it} \qquad \forall i, \forall t, \forall \upsilon. \qquad (3.10)$$

Constraint (3.1) imposes that the DAM offers should not exceed the maximum quantity of energy that can be produced by each generator, while condition (3.2) guarantees the satisfaction of the bilateral contracts needs by means of the zero-price DAM offers. Constraints (3.3) and (3.4) limit the quantity to offer on the AM for the selling and the buying bids, respectively. In particular, the maximum quantity of a selling bid on the AM is at most equal to the production capacity minus the quantity of energy already accepted on the DAM. For a buying bid, we assume that the quantity can be at most equal to the selling quantity accepted on the DAM. Moreover, in order to avoid buying and selling bids on the AM at the same period for the same unit, we have introduced the additional binary variables $\varphi_{it}^{s+}$ and $\varphi_{it}^{s+}$ and the set of constraints (3.5–3.7). $M$ is a large enough positive number.

Constraint (3.8) sets the maximum quantity that can be offered on the ASM according to the residual available capacity. Each of the condition (3.9) defines the quantity that has to be produced by each unit for each time period and under each scenario, after knowing all the market session outcomes. This quantity must be almost equal to the minimum quantity that can be produced by each unit (contraint 3.10).

Another set of constraints is referred to the minimum uptime and downtime requirements for which we introduce two auxiliary set of constants $G_i$ and $F_i$:

$$\sum_{t=1}^{G_i} (1 - U_{it}) = 0 \qquad \forall i \qquad (3.11)$$

$$\text{with} \quad G_i = \min[T, (\text{UT}_i - \text{Up}_{i0}) U_{i0}],$$

$$\sum_{j=t}^{t+\mathrm{UT}_i-1} U_{ij} \geqslant \mathrm{UT}_i\, \Delta_{it}^{+} \qquad \forall i, t = G_i + 1, \ldots, T - \mathrm{UT}_i + 1, \tag{3.12}$$

$$\sum_{j=t}^{T} (U_{ij} - \Delta_{it}^{+}) \geqslant 0 \qquad \forall i, t = T - \mathrm{UT}_i + 2, \ldots, T, \tag{3.13}$$

$$\sum_{t=1}^{F_i} U_{it} = 0 \qquad \forall i \tag{3.14}$$

$$\text{with } F_i = \min[T, (\mathrm{DT}_i - \mathrm{Down}_{i0})(1 - U_{i0})],$$

$$\sum_{j=t}^{t+\mathrm{DT}_i-1} U_{ij} \geqslant \mathrm{DT}_i\, \Delta_{it}^{-} \qquad \forall i, t = F_i + 1, \ldots, T - \mathrm{DT}_i + 1, \tag{3.15}$$

$$\sum_{j=t}^{T} (1 - U_{ij} - \Delta_{it}^{-}) \geqslant 0 \qquad \forall i, t = T - \mathrm{DT}_i + 2, \ldots, T. \tag{3.16}$$

Constraints (3.11–3.13) represent the linear expressions of minimum uptime constraints. Equation (3.11) is related to the initial status of the units. $G_i$ is, indeed, the number of initial periods during which unit $i$ must be online to meet the minimum uptime requirements. The set of condition (3.12) is used for the periods following $G_i$, and it ensures the satisfaction of the minimum uptime constraint during all the possible sets of consecutive periods of size $\mathrm{UT}_i$. Finally, the set of condition (3.13) is needed for the last $\mathrm{UT}_i - 1$ periods, i.e. if a unit is started up in one of these periods, it remains online during the remaining periods. Similarly, conditions (3.14–3.16) provide mathematical expressions for the minimum downtime limitations.

Finally, additional constraints (3.17) and (3.18) are necessary to model the start-up and shutdown status of the units and to avoid the simultaneous commitment and decommitment of each unit.

$$\Delta_{it}^{+} - \Delta_{it}^{-} = U_{it} - U_{it-1} \qquad \forall i, t = 1, \ldots, T, \tag{3.17}$$

$$\Delta_{it}^{+} + \Delta_{it}^{-} \leqslant 1 \qquad \forall i, t = 1, \ldots, T - 1. \tag{3.18}$$

All the problem's variables but the binary ones $U_{it}$, $\Delta_{it}^{+}$ and $\Delta_{it}^{-}$ are non-negative.

## 3.4 *Objective function*

The mathematical model we are proposing aims at defining a bidding strategy for a producer that considers a trade-off between profit maximization and risk minimization. For this reason, we have considered a risk–reward objective function, which is a modelling choice that is widely used in all the applicative contexts that are characterized by a high level of uncertainty. In our case, the objective function consists in a weighted sum of the expected value of the overall profits and the CVaR on the loss function (that will be explained later):

$$\max \sum_{\upsilon \in V} \theta^{\upsilon} \text{Profit}^{\upsilon} - \kappa \text{CVaR}_{\epsilon}, \tag{3.19}$$

where $\kappa$ is a user-defined trade-off parameter accounting for the risk aversion attitude, and $\epsilon$ represents the confidence level at which the Value at Risk (VaR) and the CVaR are evaluated (usually 95% or 99%).

The adoption of an effective and computationally efficient risk measure like the CVaR (or expected shortfall) is motivated by the necessity to take into account the risk due to the high uncertainty of electricity markets. Recently, the attention to risk management in the electricity field has grown and many interesting contributions in the literature can be found. In Dahlgren *et al.* (2003), after providing the state of the art of the risk assessment in power system literature, the authors present VaR and heading instruments for managing market risk for suppliers, distributors and traders. A risk assessment on local demand forecast uncertainty is carried out in Lo & Wu (2001), and a daily VaR analysis is performed on a local electricity supplier using historical imbalance settlement data in the New Electricity Trading Arrangements system. In Fusaro (1998), a risk measurement approach based on VaR and CVaR has been applied to two electricity market scenarios. In Gollmer *et al.* (2007), a risk modelling approach based on the stochastic dominance criteria has been defined for an operation and investment planning in a power generation system. In this work, we have considered the CVaR since this risk measure has recently gained a wide consideration among practitioners in many application areas (specially in financial field) because it overcomes many of the limitations of the more popular VaR (see Artzner *et al.*, 1999 and Rockafellar & Uryasev, 2000 for a detailed discussion on the adoption of CVaR). In particular, it allows to have a more accurate measure of potential losses and is much more tractable from a computational standpoint.

The overall profit for the entire planning horizon is defined as the difference between revenues and costs. The revenues depend on the clearing prices and the quantities of energy actually cleared and, thus, are not known in advance. In particular, for each scenario $\upsilon$, the total revenues, $R^{\upsilon}$, are the sum of the revenues from each market session (that we denote by $R_{\text{DAM}}$, $R_{\text{AM}}$, $R_{\text{ASM}}$, respectively) and those deriving from bilateral contracts ($R_{\text{bil}}$, which are constant) and can be expressed as follows:

$$R^{\upsilon} = R_{\text{bil}} + R_{\text{DAM}}^{p(p(\upsilon))} + R_{\text{AM}}^{p(\upsilon)} + R_{\text{ASM}}^{\upsilon} \qquad \forall \upsilon, \tag{3.20}$$

$$R_{\text{bil}} = \sum_{t=1}^{T} R_t^{\text{bil}}, \tag{3.21}$$

$$R_{\text{DAM}}^{s} = \sum_{i=1}^{I} \sum_{t=1}^{T} \lambda_{it}^{s} \gamma_{it}^{s} x_{it} \qquad \forall s, \tag{3.22}$$

$$R_{\text{AM}}^{l} = \sum_{i=1}^{I} \sum_{t=1}^{T} \mu_{it}^{l} \delta_{it}^{l+} y_{it}^{p(l)+} \qquad \forall l, \tag{3.23}$$

$$R_{\text{ASM}}^{\upsilon} = \sum_{i=1}^{I} \sum_{t=1}^{T} \varsigma_{t}^{\upsilon} \rho_{it}^{\upsilon} z_{it}^{p(\upsilon)} \qquad \forall \upsilon. \tag{3.24}$$

On the other side, the overall costs $C^{\upsilon}$ are the sum of production costs, the cost for buying energy in the AM ($C_{\text{Prod}}$ and $C_{\text{AM}}$, respectively, depend on the evolution of market outcomes) and the start-up $C_{\text{SU}}$ and shutdown $C_{\text{SD}}$ costs:

$$C^{\upsilon} = C^{\upsilon}_{\text{Prod}} + C_{\text{SU}} + C_{\text{SD}} + C^{p(\upsilon)}_{\text{AM}} \qquad \forall \upsilon, \tag{3.25}$$

$$C^{\upsilon}_{\text{Prod}} = \sum_{i=1}^{I} \sum_{t=1}^{T} \alpha_i U_{it} + \beta_i Q^{\upsilon}_{it} \qquad \forall \upsilon, \tag{3.26}$$

$$C_{\text{SU}} = \sum_{i=1}^{I} \sum_{t=1}^{T} \text{SU}_i \, \Delta^{+}_{it}, \tag{3.27}$$

$$C_{\text{SD}} = \sum_{i=1}^{I} \sum_{t=1}^{T} \text{SD}_i \, \Delta^{-}_{it}, \tag{3.28}$$

$$C^{l}_{\text{AM}} = \sum_{i=1}^{I} \sum_{t=1}^{T} \nu^{l}_{t} \delta^{l-}_{it} y^{p(l)-}_{it} \qquad \forall l. \tag{3.29}$$

We have considered linear functions for production costs (as argued in Abhyankar *et al.*, 1998) and constant cost coefficients for start-up and shutdown costs. The cost for buying energy on the AM has an expression similar to that of the AM revenues.

The possible losses under each scenario $\upsilon$ are represented by $L^{\upsilon}$. Given a certain confidence level $\epsilon$, the CVaR is defined as the expected value of losses exceeding the VaR:

$$\text{CVaR}_{\epsilon} = \text{VaR}_{\epsilon} + \frac{1}{1-\epsilon} E\{\max[L^{\upsilon} - \text{VaR}_{\epsilon}, 0]\}. \tag{3.30}$$

Since the uncertainty within the model is represented by means of finite set of scenarios, the previous definition can be linearized, using a set of auxiliary variables and constraints (see Rockafellar & Uryasev, 2000 for a detailed description of this linearization) as following:

$$\text{CVaR}_{\epsilon} = \text{VaR}_{\epsilon} + \frac{1}{1-\epsilon} \sum_{\upsilon \in V} \theta^{\upsilon} \sigma^{\upsilon}, \tag{3.31}$$

together with the following sets of constraints:

$$\sigma^{\upsilon} \geqslant L^{\upsilon} - \text{VaR} \qquad \forall \upsilon, \tag{3.32}$$

$$\sigma^{\upsilon} \geqslant 0 \qquad \forall \upsilon. \tag{3.33}$$

The overall model is a mixed-integer multistage stochastic programming problem, with linear constraints and objective function. It is important to outline that real-life instances of this problem could have very large dimensions, due in particular to the need for a sufficiently large number of scenarios for an accurate representation of the uncertainty. For this reason, it is important to adopt an efficient solution algorithm that exploits the model peculiarity and takes into account the constraints structure.

TABLE 1 *Technological characteristics of the generation units*

| | Generator 1 | Generator 2 | Generator 3 |
|---|---|---|---|
| $Q_z^{\min}$[MWh] | 0 | 0 | 0 |
| $Q_z^{\max}$[MWh] | 500 | 400 | 280 |
| $\mathrm{UT}_i$[h] | 4 | 4 | 4 |
| $\mathrm{DT}_i$[h] | 4 | 4 | 4 |
| $\mathrm{Up}_{i0}$[h] | 0 | 0 | 0 |
| $\mathrm{Down}_{i0}$[h] | 4 | 4 | 4 |
| Location zone | North | Middle north | Sardinia |

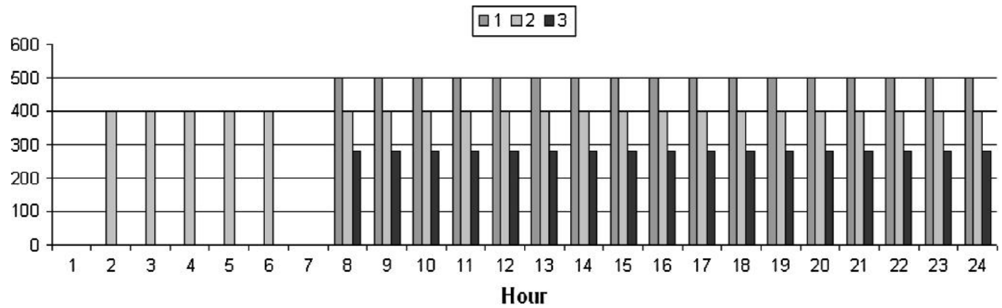


FIG. 3. Units hourly bids on the DAM [MWh].

## 4. Computational experiments

In order to validate the effectiveness of the proposed model, a set of preliminary computational experiments has been carried out. We have defined a simple test problem to show and comment the results. The starting basis of our experience is represented by a small GENCO that operates in the Italian market, with three thermoelectrical generators, whose operational characteristics are reported in Table 1.

The coefficients of the production cost function as well as the start-up and shutdown costs are the same for the three generators and assume the values $\alpha_i = 892$, $\beta_i = 14$, $\mathrm{SU}_i = 805$ and $\mathrm{SD}_i = 43$, respectively.

We have considered a time horizon of 1 day divided into intervals of 1 h each. The uncertainty has been modelled by means of a scenario tree with a 1-10-10-3 structure, i.e. the root node has 10 sons, corresponding to 10 possible DAM outcomes, each node at Stage 1 has 10 sons related to 10 possible AM outcomes and so on. The random variables have been modelled according to an analysis of historical values in the Italian market.[1] In particular, we have observed and analyzed market clearing and zonal prices of working days of January 2005. Starting from Stage 1, i.e. the DAM session, for each hour and for each zone, we have calculated the maximum and minimum values and have divided this range into 10 intervals. Each interval is represented by its mean value and has a different probability of occurrence, estimated on the basis of historical observation. Moreover, we have simulated also the percentage of bid acceptance for each zonal price, associating a higher value to a lower zonal price. The combination of simulated zonal prices and bid acceptance percentage constitute the scenarios for the DAM session.

[1] www.mercatoelettrico.org

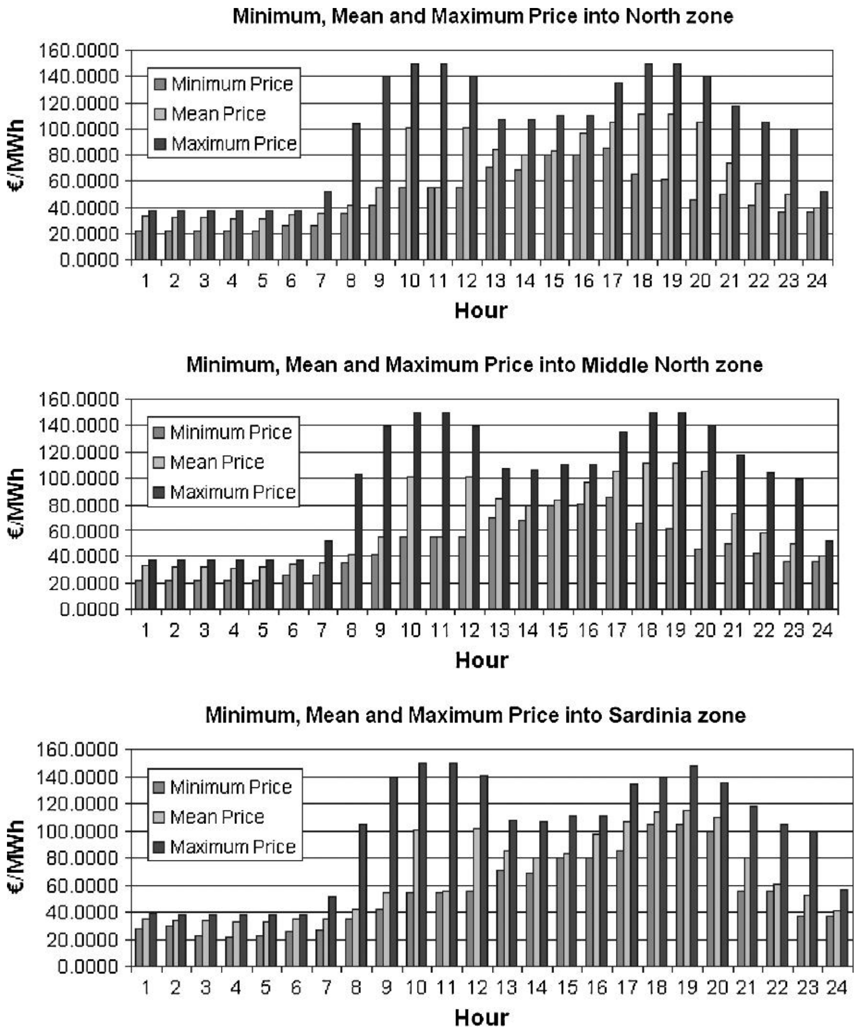


FIG. 4. Minimum, mean and maximum DAM clearing price in the the zones of north, middle north and Sardinia [Euro/MWh].

TABLE 2 *Results for confidence level* $\beta = 0.95$

| $\kappa$ | $E$[Profit] [Euro] | CVaR [Euro] |
|---|---|---|
| 1 | 1469983 | 57367.68 |
| 5 | 1434221 | 47101.84 |
| 10 | 1361978 | 37815.8 |
| 20 | 1227773 | 29700 |

A similar procedure has been implemented for the AM, generating 10 sons for each DAM scenario. However, the possibility to submit even buying bids on AM imposes to simulate not only the zonal prices but also the buying clearing price and different values of bid acceptance percentages for the different kinds of bids (buy or sell). As for the ASM, we have generated three sons for each AM

TABLE 3 *Results for confidence level* $\beta = 0.99$

| $\kappa$ | $E$[Profit] [Euro] | CVaR [Euro] |
|---|---|---|
| 1 | 1374260.42 | 59886.5 |
| 5 | 1336346.21 | 45766.27 |
| 10 | 1264161.3 | 35836.44 |
| 20 | 1136005.88 | 27417.2 |

TABLE 4 *Minimum and maximum profit for confidence level* $\beta = 0.99$

| $\kappa$ | Minimum profit | Maximum profit |
|---|---|---|
| 1 | 337571 | 2445308 |
| 5 | 337571 | 2312915 |
| 10 | 337571 | 2102096 |
| 20 | 337571 | 1829522 |

scenario, simulating a unique clearing price for all the zones and three possible values of bid acceptance percentage.

The overall scenario tree has 300 leaves, corresponding each to a particular evolution of the uncertain market outcomes. We observe that the definition of a more sophisticated scenario generation technique will be necessary for big real world applications but it is beyond the scope of this paper and may be the subject of future research. Interested readers are referred to Consigli *et al.* (2000) for a good reference on this topic.

It is also worthwhile noting the lack of general purpose solution packages for mixed-integer multi-stage stochastic programs. Such topic of research is indeed still in its infancy and only a limited number of algorithms have been proposed in the literature so far, most of which are oriented to specific applications. A non-exhaustive list includes the approaches proposed by Nowak & Römisch (2000) applied to solve the unit commitment problem, by Lulli & Sen (2004) that solve the Lot sizing problem , by Alonso-Ayuso *et al.* (2007) to solve the sequencing and scheduling problem and by Ahmed & Sahinidis (2003) that deals with the multistage capacity expansion planning. For this reason, we have implemented and solved our model by using AIMMS[2] as modelling environment and ILOG CPLEX[3] as optimization solver. The main aim is to analyze the effects of the risk aversion attitude of the GENCO on the multi-auction bidding process. For this purpose, a set of computational experiments have been conducted considering different values of risk aversion parameter $\kappa$ and confidence level $\epsilon$.

Figure 3 depicts the three units hourly bids on the DAM. The minimum, mean and maximum zonal clearing prices on the DAM for the three zones of location of the units are reported in Fig. 4. It can be underlined that when clearing price is low (off peak hours 1,...,7), in general, it is preferred not to bid with all the units and to devote the capacity for other sessions. Note also that, even though the three units are similar and differ only in their maximum capacity, bids on the DAM are different, this fact is due to the different location zones. Tables 2 and 3 report the values of expected profits and risk for different values of risk aversion parameter $\kappa$ for a confidence level of 0.95 and 0.99, respectively. Making a comparison

[2]AIMMS Optimization Modelling, Paragon Software, www.aimms.com
[3]CPLEX Optimizer, ILOG Software, www.ilog.com

between these tables, only a slight difference can be observed in the $E$[Profit] and CVaR columns, deducing thus that, for the problem under examination, confidence level reveals to be a non-fundamental parameter. The same consideration is valid for the minimum and maximum leaf profit for each unit. Indeed, since they do not vary significantly with $\beta$, they are reported in Table 4 only for $\beta = 0.99$.

Conversely, the risk aversion parameter, $\kappa$, appears to be a basic parameter. Starting to comment Table 2, a non-conservative approach, that means a low value of $\kappa$, implies a high-risk propensity (a higher CVaR value) and a more lucrative bidding strategic plan (higher expected profits). Moreover, from the analysis of Table 4, it is evident that while the minimum scenario profit, for each unit, is always the same, for different values of the risk aversion parameter, $\kappa$, on the contrary, the maximum scenario profit, for each unit, is strongly sensitive to $\kappa$. In other words, the model tries to define the optimal bidding and production strategy that hedges against all the possible scenarios. This fact could allow to a GENCO to define each time a different value of the risk version parameter according to the medium-term evolution of the market outcomes or to other strategic considerations.

## 5. Conclusion

In this paper, we present and discuss a multistage stochastic formulation for the bidding problem of a GENCO operating in a liberalized electricity market. The resulting model is an integrated tool that supports the GENCO in defining the plants schedule, the bilateral contracts satisfaction and the bidding strategies while monitoring the risk related to the random market outcomes. The necessity of defining a unit commitment-based approach has introduced into the multistage model binary variables that, together with the scenario tree representation of the uncertainty, has increased the complexity of the mathematical formulation. In this paper, we have focused on the validation of our decision approach by considering a small realistic test problem that has been solved with general purpose solvers. However, for big GENCOs, it will be necessary to develop a sophisticated scenario generation procedure and a specialized solution algorithm. These issues could be some of the directions for future investigations.

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