Optimum Transmission and Distribution Network Expansion at Several Voltage Levels through Multicommodity Network Flows

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Abstract— Network expansion is an optimization problem where investment and operating costs must be minimized while satisfying a number of security constraints. A new model consisting of two stages is proposed in this work. In the first stage a very superabundant network is defined at all voltage levels considered and in the second stage a continuous multicommodity network flow optimization process is carried out. At the optimizer only those transformers and lines to be built up, and part or all the existing ones, will carry flow bigger than zero and the ones not to be built will have zero flow. Since the problem formulation is nonconvex, it has many local optimizers. The optimization is restarted from many different initial points and the best solution obtained is retained.

Keywords—Network Expansion Planning, Transmission and Distribution, Multicommodity Network Flows, Nonlinear Network Optimization, Side Constraints

I. Introduction

The network expansion problem addressed is that of finding the lowest cost expansion of an existing transmission network in order to account for load demand growth and increasing power generation availability. The investment cost of new transmission and distribution lines and power transformers from some power source points to load points, and the operating cost expressed as estimated cost of active power losses on the existing plus new network should be minimized. Problem data are: a set of generating points with their geographical coordinates, generating capacity and voltage level, a set of load points with their geographical coordinates, load value and voltage level, a set of existing lines and transformers uniting some generation and load points, and a set of functions of change of investment cost of lines and transformers at different voltage levels with power rating. Finally there is a price to evaluate power losses, an estimation of the usage rate (in percentage of time over a year) of the transmission and distribution equipment, the interest rate to be considered and the investment pay-off time length.

The transmission network expansion problem can be formulated dinamically, with the expansion scheduled over several time steps [1,2,3], or statically [4,5,6,7]. Solving realistic versions of this problem means optimizing a large scale nonlinear mixed integer problem, which is a very difficult nonconvex mathemathical programming problem. To simplify the original problem, several approximated formulations have been adopted in the literature. The most common approach [2,3,5,6,7] is to linearize the expression of the investment and operating costs. Thus the problem can be modelized as a mixed-integer linear programming problem, where the decision variables are binary variables related to the installation or non-installation of new equipment (feeders, transformers, etc.) and continuous variables related to power flows on a dc approach. The weakness of this approach is the unrealistic formulation of the investment costs, which actually are a fixed installation cost plus a nonlinear function of the usage level, and the cost of the transmission losses, which change quadratically with power flow. A more evolved quadratic mixed integer formulation is considered in [4] , where transmission losses are a quadratic function of power flows, but substation and transformer investement and operating costs are taken as constant.

An alternative to the linear-quadratic mixed integer formulation is to avoid the use of binary decision variables through a sophisticated nonlinear formulation of the objective function, and this is the approach followed in this paper. Youssef and Hackam [1] have used this approach to formulate a dynamic model, which was used to solve the static 6-bus Garver test system [8] . They formulate a fully nonlinear approximation to the transmission investment and losses costs using an ac load flow. This continuous nonlinear formulation is, however, a nonconvex problem with multiple local minima. A procedure for overcoming this problem has been developed and presented in this work.

A common feature in the published papers is that they solve problems where there is a quite short list of potential new transmission lines provided by the utility company. The starting point in the problem solved here is more open: it is left to the program to find out the most convenient voltage level at which a given load is to be delivered, and this leaves many more decision variables to be determined. The procedure put forward is formulated for a network with existing transmission lines and transformers, but the example employed to illustrate it considers 20 load points, 4 generation points, 3 potential voltage levels, and no existing transmission lines and transformers.

Security constraints are usually considered either limiting the bus voltage magnitudes and bus swing angles [1] , or protecting the network against single line contingencies [5] .In the work presented here, special security constraints have been included: that at least two lines at the same voltage level must carry the load to a given load point, and that there should be disjoint paths from each load point to at least two different generation points. Should these security constraints not be included, a radial type network expansion would be the optimal solution.

II. Generation of a Superabundant Network at Several Voltage Levels

It will be assumed that although all loads have a specified voltage level (usually the lowest), power can be delivered to them at any voltage level equal to or higher than the load voltage. (The possibility of carrying power from low to high voltage level is ruled out). This means that at all load points all voltage levels envisaged equal or higher than the load voltage should be considered. The user may also decide which voltage levels to consider in generation sources. Thus, at each load and generation point all transformers from any voltage to all lower levels considered will be taken into account.

As for transmission lines, if any load or generation point at a given voltage level were be connected to the rest of the load or generation points at the same voltage, we would have an enormous number of variables to consider. In order to have a reasonable number of variables rules will be defined to decide which lines to take into account at each voltage level. The set of lines and transformers considered will be referred to as a superabundant network, and it should include the optimum network subset. An optimization process described later will try to find out this optimum network.

Should the procedure have to include for consideration possible station locations that are neither a load nor a generation point, it would suffice to take them as load points with a zero load at the lowest voltage level.

A. Line Generation Rules

The following algorithm to determine the superabundant network is put forward:

1) Define a set of Nv voltages to be considered

2) Follow rule 1 for the highest voltage considered

3) For each voltage level considered

  3.1) follow rule 2

  3.2) follow rule 3

  3.3) if voltage level is not the highest, follow rule 4

4) End

Rule 1: (only for the highest voltage level) Load points close enough to an existing transmission line (at the highest voltage level) are linked to that line through an input-output T connection. The critical distance δ_1 for following or not following this rule is user-defined.

Rule 2: Given a voltage level and each pair of (load or generation) nodes that have this voltage and are separated no more than a predetermined function of voltage $\delta(V)$, an ideal alignment is made between both nodes. All load nodes not yet connected at this voltage and close enough to the ideal alignment are connected by a zigzag line starting at one of the nodes of the pair and ending in the other. This operation is made for each possible pair of nodes with a suitable voltage. The critical distance δ_2 for following or not following this rule is user-defined.

Rule 3: Given a voltage level, any load point still unconnected at the voltage considered is linked to the two closest nodes already connected at this voltage.

Rule 4: (not to be applied to the highest voltage level) Given a voltage level and an existing (load or generation) node H that has a higher voltage, a closed loop of connections at the given voltage is made with a restricted set of nodes within a predetermined distance. The selection of this restricted set is as follows:

- select all nodes within a circle of radius δ_3 centered at H
- form all possible groups of nodes including node H from the nodes selected, provided that the longest distance between any pair of nodes is δ_3, and retain only the group with most nodes in it (in case of tie, discard that with the longest maximum distance)

- make the list of the arcs linking the nodes in the group chosen, provided that their distance is less than δ_4 ($\delta_4 < \delta_3$)
- with the nodes of the group and the arcs of the list find, if it exists, the largest closed polygon with the lowest perimeter that includes node H

Function $\delta(V)$ employed, giving maximum separation in terms of voltage in Rule 2, is just expressing in km the voltage in kV.

Fig. 1 shows an example of what a superabundant network could be.

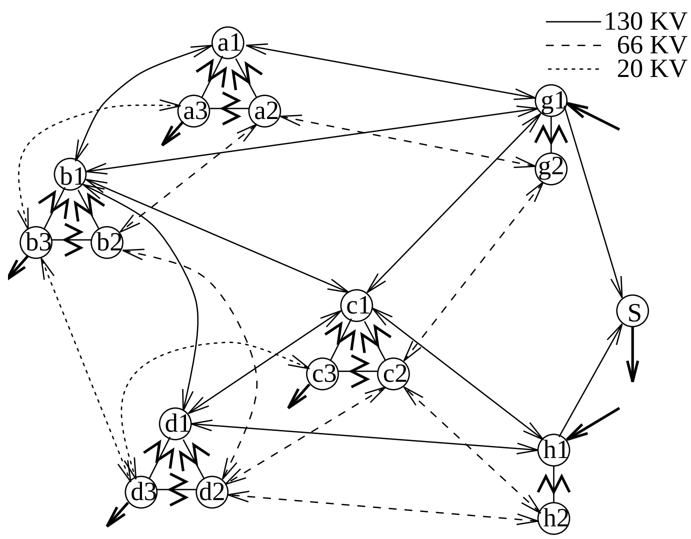

Fig. 1 Example of superabundant network from generating sources g and h to load nodes a, b, c and d at three voltage levels

III. Selection through Optimization of the Most Economical Set of Lines and Transformers

We wish to cast a network optimization problem [9] whose solution tells us which lines and transformers are to be installed and which are not. This will be done by taking a network with the same nodes as the superabundant network obtained through the procedure described, and taking as arcs its lines and transformers. The variables to be optimized will be the flows on the arcs. The arcs with zero flow at the solution are not worth building, whereas those with a flow bigger than zero are to be built. The flow on the arcs of the networks will be active power.

Some constraints will have to be imposed on the variables (power flows) in order for the solution obtained to resemble the electrical flows on the line and transformers to be installed. This is possible by considering for the constraints the parameters of the the dc aproximation to the ac electrical network.

More constraints will have to be included so as to make the solution sought satisfy a set of usual security requirements.

IV. Network Model

Given that the problem has network structure, it is worth using the well-established network codes [9] and terminology.

A. Arc Duplication

In network flow structure, arcs are directed with zero minimum flow, whereas transmission lines are not because power on them may flow in any direction. Therefore, in the network model of the superabundant electrical network to be considered, transmission lines will be represented by a pair of directed arcs, one in each direction, as in Fig 2 where a transmission line between nodes k and l is depicted. This pair of arcs will have a single investment cost and the same capacity each c_{kl}.

Flow on the arc from node k to node l will be represented by p_{kl} and p_{lk} will be the flow from l to k. In a network solution only one of the pair or none of them will carry flow.

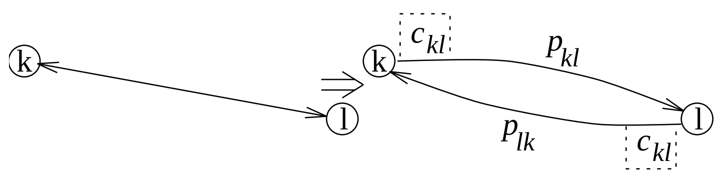

Fig. 2 Arc duplication of line uniting nodes k and l

Only one arc corresponds to a transformer since the permited flow is only from high to low voltage level.

B. Satisfaction of Kirchhoff's current and voltage laws

Kirchhoff's current and voltage laws must be satisfied by power flows if these must resemble the flows on the real lines and transformers to install.

In a dc network model power and current flows measured in per unit (p.u.) at base voltage coincide. Thus balance of power flow at the nodes of the network ensures the satisfaction of Kirchhoff's current law.

Kirchhoff's voltage law must be satisfied around all loops of the electrical network. (Not in all loops of the superabundant network since only a few of its arcs are to become the future electrical network). In fact it is enough to impose it on all *basic* loops in the transmission network. Since it is not known a priori which of the arcs of the superabundant network will be part of the future electrical network, Kirchhoff's voltage law will be imposed on the existing loops (with arcs with flow bigger than zero) of a previous solution and the problem will be reoptimized taking into account these constraints.

Let x_{kl} be the p.u. reactance of the transmission line (or transformer) corresponding to the arc going from node k to node l, and let p_{kl} be the power flow from k to l. The voltage drop along arc k–l can then be expressed as $x_{kl}p_{kl}$. Thus, the expression of Kirchhoff's voltage law is:

$$\sum_{(k,l)\in \text{loop} \quad j} x_{kl}p_{kl} = 0 \quad \text{for all basic loops} \quad j \tag{1}$$

Considering the examples in Fig. 3, we have that the satisfaction of Kirchhoff's current law in node $d3$ would mean that $p_{b3d3} - p_{d3b3} + p_{c3d3} - p_{d3c3} + p_{d1d3} + p_{d2d3} = l_d$. Kirchhoff's voltage law for the loop in Fig. 3b), taking into account double arcs, would be:

$$(p_{d1c1} - p_{c1d1})x_{d1c1} + (p_{c1h1} - p_{h1c1})x_{c1h1}$$
$$+p_{h1h2}x_{h1h2} + (p_{h2d2} - p_{d2h2})x_{h2d2} - p_{d1d2}x_{d1d2} = 0$$

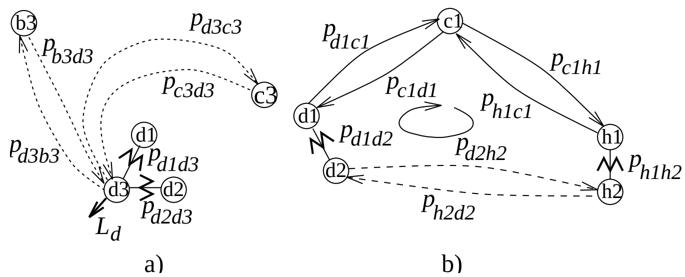

Fig. 3. a) Example of arcs for application of Kirchhoff's current law at node d3

b) Example of loop for application of Kirchhoff's voltage law

Kirchhoff's voltage law in the dc network formulation is a side constraint [9] .

C. Formulation of a Continuous Objective Function

Network expansion problems have traditionally been addressed with combinatorial optimization techniques using binary variables y_{kl}, zero or one, associated to the decision not to construct or to construct the link uniting nodes k and l of capacity c_{kl} and investment cost C_{kl}. However, when quadratic transmission losses have to be taken into account, the relaxation of the integrity (0 or 1) condition of variables y_{kl} leads to optimizers with very few relaxed variables either at 0 or 1, which makes the solution through these techniques very complicated.

A possible way of solving this problem through continuous minimization would be to consider a function to be minimized such as that of Fig. 4:

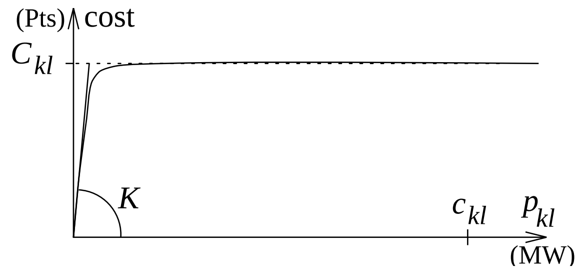

Fig. 4 Exponential function approximating investment cost with respect to power carried

$$
\left.\begin{aligned} C_{kl}\left[1-\exp^{-\frac{K}{C_{kl}}(p_{kl}+p_{lk})}\right] \\ 0 \le p_{kl} \le c_{kl} \\ 0 \le p_{lk} \le c_{kl} \end{aligned}\right\} \tag{2}
$$

where C_{kl} is the cost of the line or transformer, K is the (quite steep) slope of the exponential at the origin, p_{kl} and p_{lk} are the power flows from k to l and form l to k respectively, and c_{kl} is the line or transformer power rate.

This function has an interesting feature: should the power carried on link k–l be big, the derivative is practically zero, but should the power carried be small, the derivative gets to be almost K, which is big, so its construction is discouraged. Unfortunately, an objective function that includes one such exponential function for each link under consideration is nonconvex with many local minimizers, so special techniques will have to be employed to get to the global optimum.

The cost function usually taken into account is not as that of Fig. 4, because depending on the power to be carried, many line structures (single circuit, double circuit, etc.) may be envisaged. The real cost function is then as that of Fig. 5a), and it can be approximated by a function such as that of Fig. 5b), which can be formulated as a linear function plus an exponential like (2):

$$
B_{kl}(p_{kl}+p_{lk}) + C_{kl}\left[1-\exp^{-\frac{K}{C_{kl}}(p_{kl}+p_{lk})}\right] \tag{3}
$$

where B_{kl} is the linear term of the investment cost.

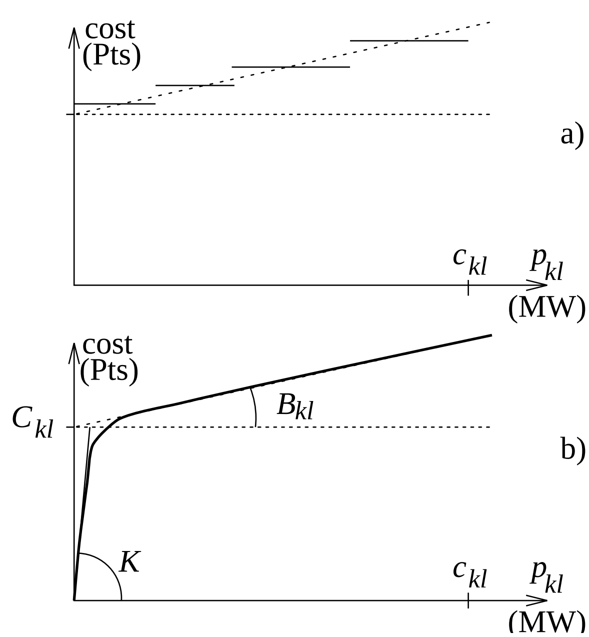

Fig. 5. a) Real cost function of transmission line or transformer for several structural options, in terms of power to be carried

b) Continuous approximation to real cost function

D. Evaluation of Transmission Losses

Transmission losses mean an operating cost that must be minimized, together with the investment in new lines and transformers. Usually the transmission loss evaluation corresponding to a year and the annual straight line investment pay-off are added up in the objective function to be minimized.

In the dc network model, current and power in p.u. are equivalent. Thus the value of yearly real power losses on the line or transformer corresponding to arcs uniting nodes k and l is:

$$\lambda r_{kl}(p_{kl} - p_{lk})^2 \tag{4}$$

where λ is the product of loss value per base power loss times the rate of in-service time of transmission lines and transformers, and r_{kl} is the p.u. resistence of the transmission line or transformer considered.

V. Multicommodity Network Flow Model

Many of the security constraint to be taken into account can not be imposed with the formulation described so far, but can be easily expressed through the following multicommodity [9,10] model.

The power corresponding to each load will be considered a different commodity, so there will be as many commodities flowing on the network as load nodes (Nl). Instead of having a nonlinear single commodity network flow problem with side constraints [11] we will have to solve a nonlinear multicommodity network flow problem with side constraints [10] . The different commodities will be identified in our notation by a superscript preceded by an opening parenthesis, e.g. in the network of Fig. 1 (with loads at nodes a, b, c and d) the flow on any arc from node k to node l would be $p_{kl} = p^{(a}{}_{kl} + p^{(b}{}_{kl} + p^{(c}{}_{kl} + p^{(d}{}_{kl}$. According to the classical multicommodity formulation [9] , there will be a specific arc capacity for each commodity $c^{(i}_{kl}$ for the i^{th} commodity: $p^{(i}_{kl} \le c^{(i}_{kl}$, and a mutual capacity constraint for each arc, which coincides with the arc capacity considered all along, so that, assuming that there are Nl commodities (loads) we have that: $\sum_{i=1}^{Nl} p^{(i}_{kl} \le c_{kl}$.

A. Load Delivery through More Than One Line

Security rules impose that power be delivered to a given load node through more than one line. To achieve this, it is enough to impose that for the i^{th} commodity, which corresponds to the i^{th} load —of value l_i —, all power carried on any line is strictly less than l_i, so that at least two lines will be necessary to carry power l_i to load node i. In the programs developed this is usually expressed as:

$$\left.\begin{aligned} 0 \le p^{(i}_{kl} \le c^{(i}_{kl} = .7\, l_i \\ 0 \le p^{(i}_{lk} \le c^{(i}_{kl} = .7\, l_i \end{aligned}\right\} \tag{5}$$

This limit is only imposed on transmission lines, but not on transformers, where $c^{(i}_{jm\text{-}jn} = c_{jm\text{-}jn}$ for a transformer at the j^{th} station from voltage level m to voltage level n.

B. Load Supply from More Than One Generating Source

A common practice is to design network expansion so that there should be more than one path, with no common arcs, from any load node to generation sources, and so that, should there be a line failure in an arc of one of the paths, there would be at least an alternative path to deliver power to the load node. To achieve this, there will be a positive injection $P_g^{(i}$ of the i^{th} commodity at each generation point g, which corresponds to the i^{th} load —of value l_i —, of strictly less than l_i. In the programs developed the authors have used:

$$P_g^{(i} = .7\, l_i \tag{6}$$

In order for total generation at the generation source g to be less than the generating capacity $\overline{P}_g$, a (linear) side constraint must be imposed at each generation point so that all the outcoming power —not going to the sink node— at all voltage levels of all commodities (loads) is less than $\overline{P}_g$:

$$\sum_{i=1}^{Nl} \sum_{m=1}^{Nv} \sum_{k \in Igm} p_{gm\text{-}k}^{(i} \leq \overline{P}_g \quad (g = 1, \ldots, Ng) \tag{7}$$

where Igm is the set of nodes —excluding de sink S — connected to node g at the m^{th} voltage level.

The sink S will receive the extra generating capacity of all commodities:

$$\sum_{g=1}^{Ng} P_{g1}^{(i} - l_i \quad (i = 1, \ldots, Nl) \tag{8}$$

(it should be noted that generation injection in the network takes place at the highest voltage level of the generation point, and that to simplify notation it has been assumed that the highest voltage is the same in all generation points and that this is the highest voltage level in the network; that is why notation $P_{g1}^{(i}$ employing $g1$ (1st. or highest level) has been employed).

C. Voltage Uniformity in the Delivery of a Given Load

It is desired that the delivery of load l_i to node i be made by more than one line, but at the same voltage level. The reason is that capacities of lines usually increase with voltage level, so in case of delivery by two lines at different voltage levels, should the high voltage line fail, the lower voltage line would not be usually capable of carrying load l_i. (Notice that we could avoid this by using .51 instead of .7 in (5), but this would leave little room for optimization). In order for delivery of the i^{th} load to be made at only one of the Nv possible voltage levels, a penalty product term such as:

$$\psi \Pi_{m=1}^{Nv} (l_i - \sum_{k \in Iim} p_{k\text{-}im}^{(i}) \tag{9}$$

is added to the objective function, where ψ is a big positive penalty term and Iim is the set of nodes at voltage level m, connected to point i through a transmission line (at voltage level m). Should all power l_i arrive in node i at voltage level m, $(l_i - \sum_{k \in Iim} p_{k\text{-}im}^{(i})$ would be zero and so would at (9), but if only part of l_i arrives at node i at voltage level m, $(l_i - \sum_{k \in Iim} p_{k\text{-}im}^{(i})$ would be bigger than zero and less than l_i, at least another of the multiplicands of (9), at a different voltage level, would also be strictly between zero and l_i, and the rest of the multiplicands would be l_i. Thus the miminum of the penalty term (9) is zero when delivery of load is made at only one voltage level.

D. Avoiding Bottleneck Stations

Constraints (5), (6) and (7) and penalty terms (9) may permit that the paths from different generation points to a given load i join at a given intermediate node j, whereby in case of a busbar failure at j there could be no possibility of delivering power to i. To make sure that there are no joints in the paths from the generating points to the load node i it is enough to include the following (linear) side constraints expressing that all power of the i^{th} commodity (i^{th} load) coming out of station j is strictly less than l_i:

$$\sum_{m=1}^{Nv} \sum_{k \in Ijm} p_{jm\text{-}k}^{(i} \leq .95\, l_i \qquad (i = 1, \ldots, Nl),\ (j = 1, \ldots, Nl,\ j \neq i) \tag{10}$$

where Ijm is the set of points at voltage level m connected to point j through transmission lines, and the .95 constant is just to express that there should be at least another point where the i^{th} commodity arrives and leaves out with a flow bigger than zero, thus avoiding the possibility of a bottleneck station in j for load l_i.

There are $Nl \times (Nl-1)$ such constraints and they have not been included in the implementation made because so far the solutions obtained have had no conflicting bottleneck points, but they could be readily incorporated if necessary.

VI. Problem Formulation

Taking into account that $p_{kl} = \sum_{i=1}^{Nl} p_{kl}^{(i}$ and $(p_{lk} = \sum_{i=1}^{Nl} p_{lk}^{(i})$, the objective function to be minimized is

$$\min \sum_{(k,l) \in Ja} \left\{ B_{kl}(p_{kl} + p_{lk}) + C_{kl}\left[1 - \exp^{-\frac{K}{C_{kl}}(p_{kl}+p_{lk})}\right] \right\} + \lambda \sum_{(k,l) \in Ia} r_{kl}(p_{kl} - p_{lk})^2 + \psi \sum_{i=1}^{Nl} \Pi_{m=1}^{Nv} (l_i - \sum_{k \in Iim} p_{k\text{-}im}^{(i}) \tag{11}$$

which includes the investment cost, the losses and the voltage uniformity penalization, and where Ja is the set of pairs of nodes that are the ends of all new lines and transformers of the superabundant network and Ia is the same set but for all lines and transformers (existing and new).

The constraints to take into account are:

$$\sum_{k \in Iim} (p_{k\text{-}im}^{(i} - p_{im\text{-}k}^{(i}) + \sum_{n=1}^{m-1} p_{in\text{-}im}^{(i} - \sum_{n=m+1}^{Nv} p_{im\text{-}in}^{(i} = l_{mi} \qquad m = 1, \ldots, Nv \quad i = 1, \ldots, Nl \tag{12}$$

which is the balance equation of the i^{th} commodity at the i^{th} load node at voltage level m. l_{mi} would be zero if the load at the i^{th} node is not at the m^{th} voltage level.

$$\sum_{k \in Iim} (p_{k\text{-}im}^{(j} - p_{im\text{-}k}^{(j}) + \sum_{n=1}^{m-1} p_{in\text{-}im}^{(j} - \sum_{n=m+1}^{Nv} p_{im\text{-}in}^{(j} = 0 \qquad m = 1, \ldots, Nv \quad i = 1, \ldots, Nl \quad j = 1, \ldots, Nl,\ j \neq i \tag{13}$$

which is the balance equation of the $j^{\rm th}$ commodity at the $i^{\rm th}$ load node at voltage level m.

$$\sum_{k \in Ig1} (p^{(i}_{g1\text{-}k} - p^{(i}_{k\text{-}g1}) + \sum_{n=2}^{Nv} p^{(i}_{g1\text{-}gn} + p^{(i}_{g1\text{-}S} = P^{(i}_{g1} \qquad i = 1, \ldots, Nl \quad g = 1, \ldots, Ng \tag{14}$$

which is the balance equation of the $i^{\rm th}$ commodity at the $g^{\rm th}$ generation node, assuming that generation injection takes place at the highest voltage level (the 1st.)

$$\sum_{k \in Igm} (p^{(i}_{gm\text{-}k} - p^{(i}_{k\text{-}gm}) + \sum_{n=m+1}^{Nv} p^{(i}_{gm\text{-}gn} - \sum_{n=1}^{m-1} p^{(i}_{gn\text{-}gm} = 0 \qquad i = 1, \ldots, Nl \quad m = 2, \ldots, Nv \quad g = 1, \ldots, Ng \tag{15}$$

which is the balance equation of the $i^{\rm th}$ commodity at the $g^{\rm th}$ generation node at voltage levels lower than the highest (1st.)

Constraint (8), the sink balance, completes the network balance equations.

Single commodity limits to be imposed on arcs corresponding to lines and to transformers are:

$$\left.\begin{array}{l} 0 \le p^{(i}_{kl} \le .7\, l_i \\ 0 \le p^{(i}_{lk} \le .7\, l_i \end{array}\right\} \quad (k,l) \in Il \quad i = 1, \ldots, Nl \tag{16}$$

$$0 \le p^{(i}_{jm\text{-}jn} \le c_{jm\text{-}jn} \quad j \in In \quad i = 1, \ldots, Nl \qquad m = 1, \ldots, Nv-1 \quad n = 2, \ldots, Nv \quad m < n \tag{17}$$

where Il is the set of pairs of indices of line ends at all voltages and In is the set of indices of all load and generation points.

Multicommodity mutual capacity constraints to be imposed on arcs corresponding to lines and to transformers are:

$$\left.\begin{array}{l} \displaystyle\sum_{i=1}^{Nl} p^{(i}_{kl} \le c_{kl} \\ \displaystyle\sum_{i=1}^{Nl} p^{(i}_{lk} \le c_{kl} \end{array}\right\} \quad (k,l) \in Il \tag{18}$$

$$\sum_{i=1}^{Nl} p^{(i}_{jm\text{-}jn} \le c_{jm\text{-}jn} \quad j \in In \qquad m = 1, \ldots, Nv-1 \quad n = 2, \ldots, Nv \quad m < n \tag{19}$$

Finally, side constraints (7) must be always included and those corresponding to Kirchhoff's voltage law (1) are to be included in the reoptimizations for the basic loops found in the previous optimization result.

VII. Computational Implementation

It was mentioned in Section IV that the continuous formulation proposed was not convex (due to the exponential terms). Optimization therefore leads to a local minimizer, which may change depending on the initial point fed into the minimization code employed.

The procedure followed to get closer to the global optimizer is to run the optimization many times starting from different initial points. The initial points that have been employed are solutions to simpler problems of the same multicommodity variables that are subject to the same constraints as the problem formulated.

The objective function employed to generate an initial and feasible point is:

$$\min \sum_{(k,l)\in Ja} \Big\{ B_{kl}(p_{kl}+p_{lk}) + A_{kl}(p_{kl}+p_{lk}) \Big\} + \lambda \sum_{(k,l)\in Ia} r_{kl}(p_{kl}-p_{lk})^2 \tag{20}$$

which includes a linearized investment cost and the losses, and where A_{kl}, $(k,l)\in Ja$ are constant coefficients between 0 and K generated randomly.

The constraints taken into account are (12), (13), (14), (15), (8), (16), (17), (18), (19) and (7)

To have a different initial point to be fed in the minimization of (11) subject to the constraints, a new set of random coefficients A_{kl}, $(k,l)\in Ja$ is generated, and with them (20) is minimized subject to the constraints. Thus, the algorithm employed is:

0) Set *bestcost* to infinity and initialize *counter* to 0

1) *counter*:=*counter*+1 and if *counter* exceeds limit STOP

2) generate a new set of random coefficients A_{kl}, $(k,l)\in Ja$

3) minimize (20) subject to (7), (8) and (12) through (19)

4) taking as initial point the solution obtained in 3), minimize (11) subject to (7), (8) and (12) through (19) and put the minimum objective value obtained in *currentcost*

5) compare *currentcost* with *bestcost*

   5.1) if *currentcost*$\geq$*bestcost* go to 1)

   5.2) otherwise

      5.2.1) look for loops in current solution and determine basic loops

      5.2.2) minimize (11) subject to (7), (8), (12) through (19) and (1) and put the minimum objective value obtained in *currentcost*

      5.2.3) Compare *currentcost* with *bestcost*

         5.2.3.1) if *currentcost*$\geq$*bestcost* go to 1)

         5.2.3.2) otherwise

            5.2.3.2.1) *bestcost*:=*currentcost*

            5.2.3.2.2) put current solution in file *Bestpoint*

            5.2.3.2.3) go to 1)

At the end of this process we will have in the file *Bestpoint* the best solution obtained with cost *bestcost*.

VIII. Computational Results and Case Example

The algorithm for generating a superabundant network, described in Section II, has been coded and the algorithm of Section VII with the formulation of Section VI has been coded with a specialised nonlinear multicommodity network flow code with side constraints [10] and also with the general purpose nonlinear optimization code Minos 5.3 [13,14].

To test the procedure put forward, a real transmission and distribution expansion problem has been solved. This problem considers 20 load points, 4 generation points and 3 voltage levels (130, 66 and 20kV), and its data are in Tables I through IV. Prices are in Spanish currency (Pts), the interest rate considered is 10%, the pay-off period for investment 40 years, the price for power losses Pts9/KWh and the utilization rate 4500h per year. No existing lines have been considered.

Fig. 6 shows the superabundant network obtained with the algorithm of Section II using δ_1=4km, δ_2=25km, δ_3=50km and δ_4=45km. The $\delta(V)$ function employed has been $\delta(x\text{ kV}) = x$ km. The superabundant network has 73 nodes, 362 arcs, 20 commodities, 4 side constraints (7). It has a total of 7240 (multicommodity) variables (lines are duplicated).

Fig. 7 shows the results obtained, where 13 extra side constraints (1) were introduced. Its cost is Pts 1240.94×10^6. The solution has been obtained after 250 cycles of the algorithm of Section VII (having found the best point at cycle 30). In this solution load points A,C,F,J,P and Q have their power delivered at 130 kV, load point E at 66 kV and the rest at 20 kV. Numbers by the lines indicate power carried in MW. Of the pair of numbers inside parentheses, the first means power in the direction of the arrow, and (for other commodities) the second means power in the oposite direction.

Execution times of this process on a Sun Sparc 10-41 workstation are long. Typically, taking as a sample cycle 30, which provided the solution displayed, we have that using the Minos code [13] it took 555 iterations (27.27s) to solve step 3), 1113 iterations (64.46s) were required in step 4) to get to the solution without Kirchhoff's voltage law, and in step 5.2.2) (solution with Kirchhoff's voltage law) 4671 extra iterations (314.49s) were spent. It is difficult to carry out an exact comparison between the specialized codes [12] and Minos because, as the problem solved is nonconvex, different programs reach different solution points. For step 4) in one of the cycles, the specialised code required 738 iterations (8.97 s) whereas Minos needed 733 iterations (50.6 s). Therefore, the number of iterations is similar but the time per iteration is much less with the specialised code than with Minos.

IX. Conclusions

A new procedure for solving the network expansion planning problem at several voltage levels has been presented. It is based on the repeated optimization from different initial points of a continuous investment function of power flow. The problem formulation includes security

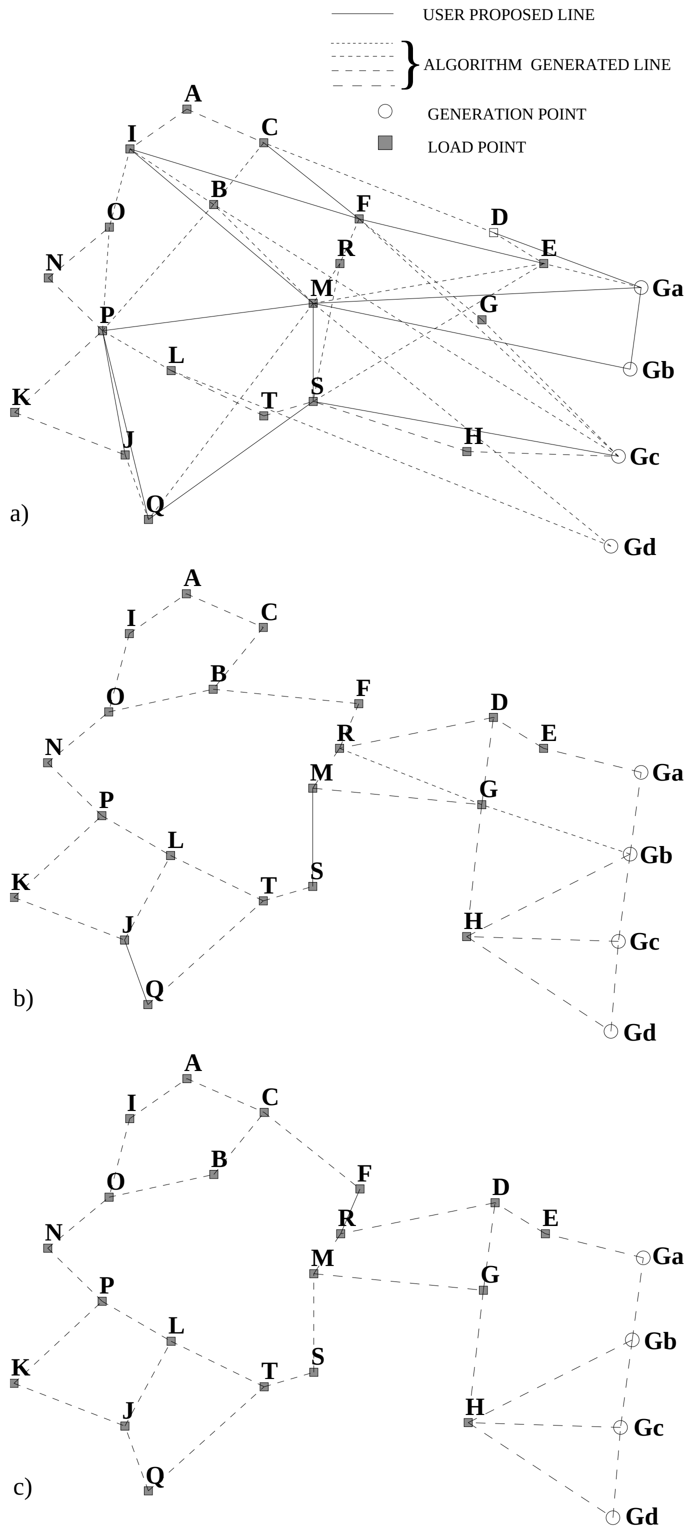

Fig. 6. a) Superabundant network of example at 130kV

b) Superabundant network of example at 66kV

c) Superabundant network of example at 20kV

Table I. Characteristics of load points.

node	load MW	kV	x km	y km	node	load MW	kV	x km	y km
A	10	20	48.5	11.0	K	3	20	22.6	73.2
B	3	20	52.5	30.6	L	5	20	46.1	64.6
C	2	20	60.0	18.0	M	5	20	67.4	50.8
D	2	20	94.5	36.4	N	2	20	27.7	45.6
E	4	20	102.0	42.7	O	2	20	36.9	35.2
F	5	20	74.3	33.5	P	9	20	35.8	56.5
G	2	20	92.8	54.2	Q	20	20	42.7	95.1
H	1.5	20	90.5	81.2	R	3	20	71.5	42.7
I	4	20	39.9	19.2	S	1	20	67.4	70.9
J	6	20	39.2	81.8	T	4	20	60.0	73.8

Table II. Characteristics of generation points.

node	Max. Capacity MW	kV	x km	y km
G_a	60	130	116.69	47.64
G_b	90	130	114.97	64.33
G_c	120	130	113.24	82.17
G_d	120	130	112.09	100.58

Table III. Characteristics of lines.

kV	R Ω/km	X Ω/km	Cap. A	10^6Pts/km= $a_0+a_1p_{kl}$(MW) a_0	a_1	Connection cost 10^6Pts
130	0.307	0.43	1140	8.5	0.067	68.9
66	0.307	0.41	1140	4.19	0.069	27.7
20	0.307	0.39	1140	2.7	0.14	20.0

Table IV. Characteristics of transformers.

High kV	Low kV	R (p.u.)	X (p.u.)	10^6Pts/km= $a_0+a_1p_{kl}$(MW) a_0	a_1
130	66	0.0063	0.1250	87.3	1.2
130	20	0.0120	0.4945	107.91	1.2
66	20	0.0139	0.2780	43.78	0.56

constraints expressed as the requirement of having to deliver power to each load through two or more lines at the same voltage level and that of having at least two disjoint paths from each load to different generation points.

Nonlinear multicommodity network flows with linear side constraints can adequately model this problem and proofs have been made with a specialized code and with a general purpose nonlinear optimization code.

Results have been reported for a quite large real example, and the results obtained confirm that a solution subject to the constraints expressed exists and that it can be obtained using the procedure put forward.

X. Acknoledgements

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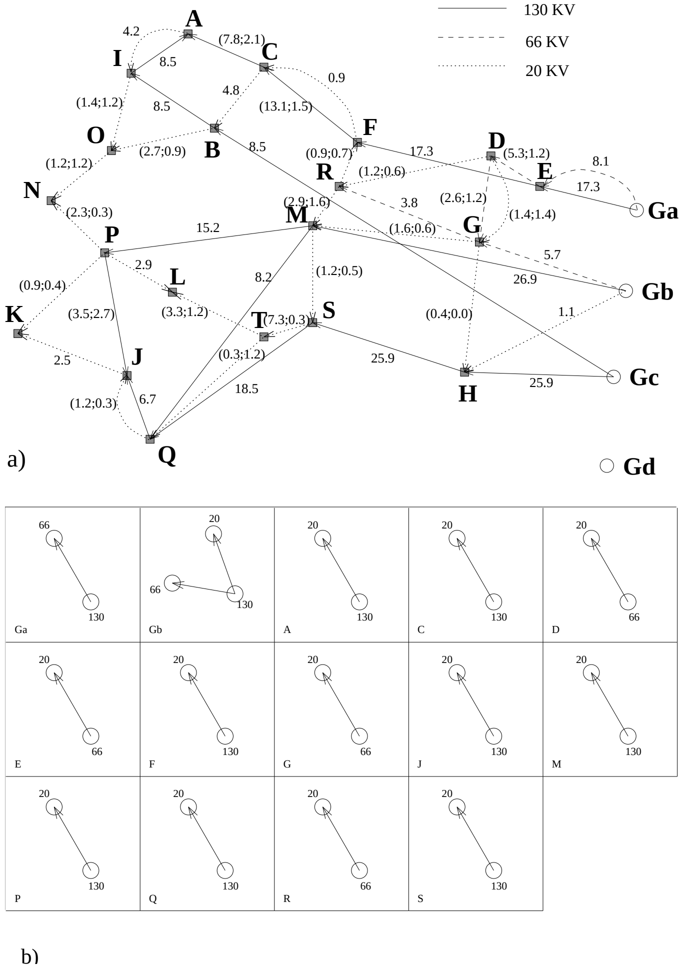

Fig. 7. a) Solution network of example at 130, 66 and 20kV

b) Transformers of solution network of example.

XI. Glossary of Symbols

A_{kl} random linear investment cost of new transmission line or new transformer between nodes k and l

B_{kl} linear investment cost of new transmission line or new transformer to be installed between nodes k and l

c_{kl} capacity of existing or new transmission line or new transformer between nodes k and l

C_{kl} basic investment cost of new transmission line or new transformer to be installed between nodes k and l

$(i$ (supraindex) indication of association to i^{th} load

Ia set of pairs of nodes that are the ends of all lines and transformers (existing and new)

$Ig1$ set of points at voltage level 1 (the highest) connected to generation point g through transmission lines

Igm set of nodes —excluding de sink S — connected to node g at the m^{th} voltage level

Ijm set of points at voltage level m connected to point j through transmission lines

Il set of pairs of indices of line ends at all voltages

jm-jn (subindex) indication of transformer at the j^{th} point from voltage level m to voltage level n

In set of indices of all load and generation points

Ja set of pairs of nodes that are the ends of all new lines and transformers of the superabundant network

K slope (big) of exponential function at the origin

l_i load at the i^{th} load point

Ng number of generation points

Nl number of load points

Nv number of voltage levels considered

$P_g^{(i}$ power generated at generation point g corresponding to the i^{th} load

$\overline{P}_g$ maximum generating capacity of generation point g

$p_{kl}^{(i}$ power of commodity corresponding to the i^{th} load carried from node k to node l

r_{kl},x_{kl} resistence and reactance of transmission line or transformer between nodes k and l

$\delta_{1,2,3,4}$ distances in line generation rules

$\delta(V)$ distance in terms of voltage level in line generation rules

λ product of loss value times rate of in service time

ψ big positive penalty term for voltage uniformity in the delivery of a given load

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