8 RELIABILITY

Systems Reliability

A system consists of components which determine whether or not it will work. There are various types of configurations of the components in different systems.

- **Series System**
  This is a system in which all the components are in series and they all have to work for the system to work. If one component fails, the system fails.

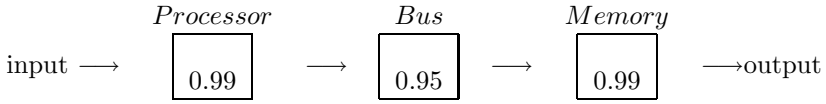
- **Parallel System**
  This is a system that will fail only if they all fail.

- **Series-Parallel System**
  This is a system where some of the components in series are replicated in parallel.

8.1 Reliability of Series System

Examples

1. A simple computer consists of a processor, a bus and a memory. The computer will work only if all three are functioning correctly. The probability that the processor is functioning is 0.99, that the bus is functioning 0.95, and that the memory is functioning is 0.99 .

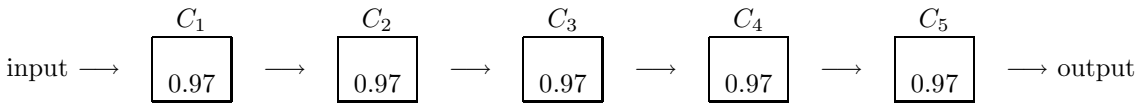


The probability that the computer will work is:

$$Rel = .99 \times .95 \times .99 = 0.893475$$

So even though all the components have above 95% or more reliability, the overall Reliability of the computer is less that 90%.

2. A system consists of 5 components in series each having a reliability of 0.97. What is the reliability of the system?



$$Rel = 0.97^5 = 0.86$$

With series systems, reliability decreases as the number of components increases.

With 6 components in series;

$$Rel = 0.97^6 = 0.832972$$

With 7 components; $Rel = 0.97^7 = 0.8079828$

3. An electronic product contains 100 integrated circuits. The probability that any integrated circuit is defective is .001 and the integrated circuits are independent. The product operates only if all the integrated circuits are operational. What is the probability that the product is operational?

   Solution:

   The probability that any component is functioning is .999. Since the product operates only if all 100 components are operational, the probability that the 100 components are functioning is:

   $$Rel = .999^{100}$$

   obtained in *R* with

```
 .999**100
[1] 0.9047921
```

   The reliability is just over 90% even though each component has a reliability of 99.9%.

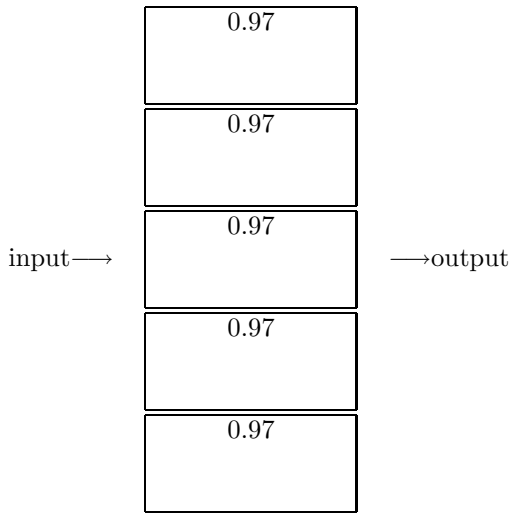
Bearing in mind that computing and electrical systems have hundreds or thousands of components, a series formation on its own will never be sufficiently reliable, no matter how high the individual component reliability is. The components need to be backed up in parallel.

8.2 Reliability of a Parallel System

Systems with parallel structure have built in redundancy. Components are backed up, and the system will work even if some of these cease to function.

Examples

1. A system consists of 5 components in parallel. If each component has a reliability of 0.97, what is the overall reliability of the system?



System will function provided at least one of the 5 components works:

$$Rel = \text{P(At least one component is functioning)}$$

Taking the complementary approach,

P(at least one component functioning) = 1- P(all components fail).

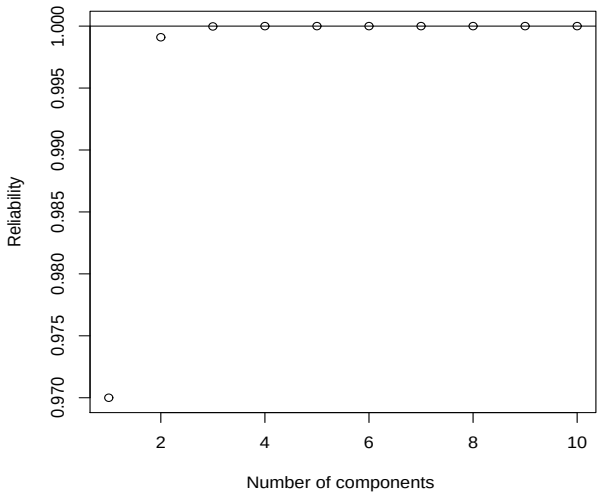
Therefore

$$Rel = 1 - (0.03)^5 \approx 1.00000$$

With parallel systems the "law of diminishing returns" operates:

With 2 components $Rel = 1 - 0.03^2 = 0.9991$
With 3 components $Rel = 1 - 0.03^3 = 0.999973$
With 4 components $Rel = 1 - 0.03^4 = 0.9999992$

Reliability of a parallel system with increasing number of components: component reliability $= .97$




```
k<-1:10
p <- .97
plot (k,  1-(1-p)^k , xlab = "Number of components ",
ylab = "Reliability")
abline(1,0)
```

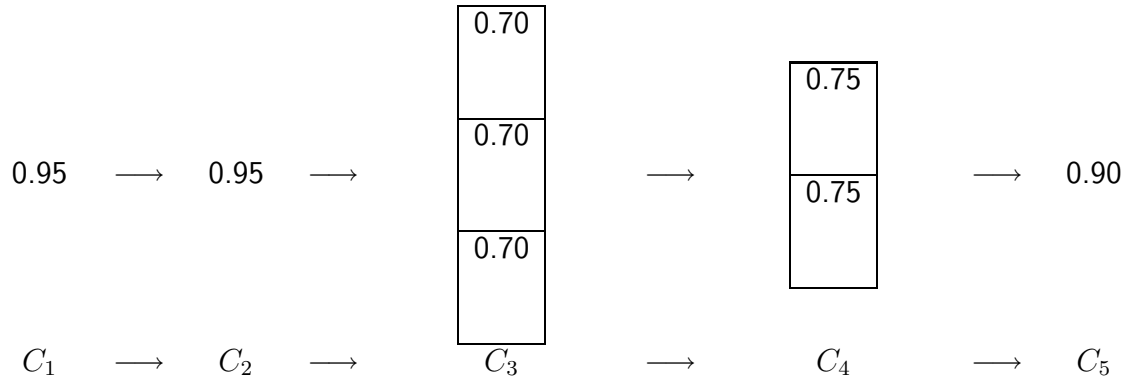
Series-Parallel Systems

Examples

1. Consider a system with 5 kinds of component, with reliabilities
   - component 1 : 0.95,
   - component 2 : 0.95,
   - component 3 : 0.70,
   - component 4 : 0.75,
   - component 5 : 0.90.

Because of the low reliability of the third and fourth components, they are replicated; the system contains 3 of the third component and 2 of the fourth component.

The System:



$P(C_1)$ × P(C_2) × P(at least 1 C_3) × P(at least 1 C_4) × P(C_5)

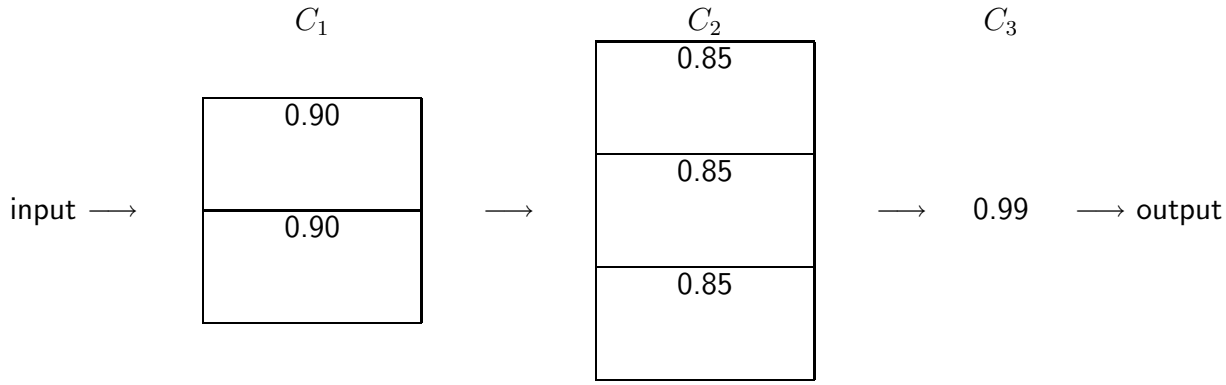
$$Rel = .95 \times .95 \times (1 - .3^3) \times (1 - .25^2) \times .9$$

In *R*

```
.95*.95*(1-.3^3) * (1- .25^2) *.9
[1] 0.7409243
```

Examples

2. The following system operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume the devices fail independently.



Reliability: P(at least 1 of 2 C_1) $\times$ P(at least 1 of 3 C_2) $\times$ $P(C_3)$

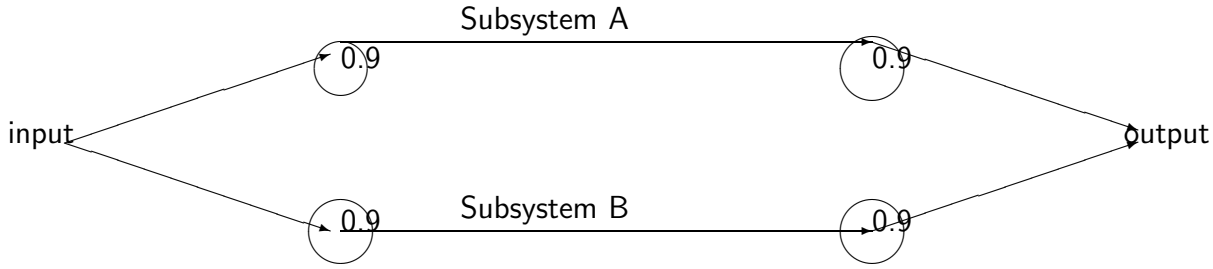
$1 - .1^2$ $\times$ $1 - .15^3$ $\times$ .99

In *R*

```
> (1-.2^2)* (1-.15^3)*.99
[1] 0.9471924
```

Examples

3. The following diagram is a representation of a system that comprises two subsystems operating in parallel.



The system operates if at least one of subsystem A operates.

$$Rel = P(\text{at least one subsystem works})$$

Equivalently

$$Rel = \text{1- } P(\text{both subsystems fail})$$

$P(\text{Subsystem A fails}) = 1 - .9^2$.

$P(\text{Subsystem B fails}) = 1 - .9^2$.

Overall system will operate if at least one of these subsystems works.

$$Rel = P(\text{at least one subsystem works})$$

Equivalently

$$Rel = \text{1- } P(\text{both subsystems fail})$$

$$Rel = 1 - (1 - .9^2)^2 = 0.9639$$

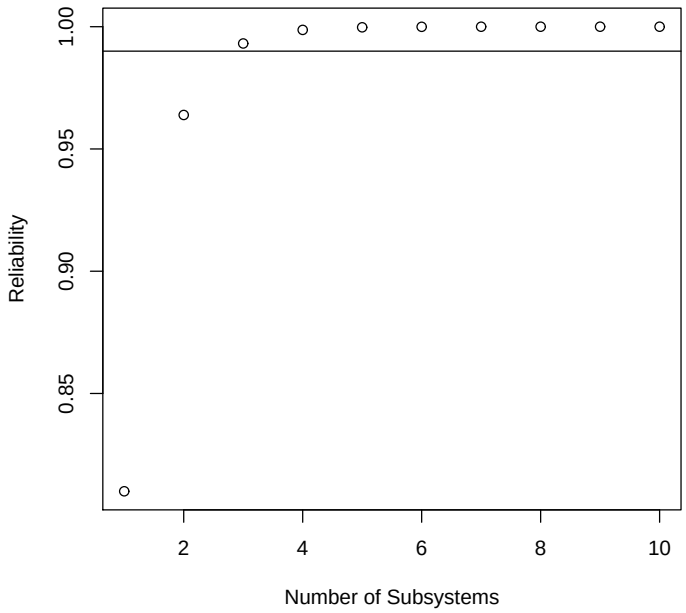
Generally with k subsystems, the reliability is

$$Rel = 1 - (1 - .9^2)^k$$

We can examine this with different values of k

```
k<-1:10
plot(k, 1-(1-.9^2)^k, xlab = "Number of Subsystems", ylab = "Reliability")
abline(.99, 0)
```

calculates and plots the reliability for various levels of backup:



Summary

The reliability of a system, i.e. the probability that it is functioning properly depends on

- the reliability of each of its the components
- the type of system

Reliability with Series Systems

The problem with series systems is that reliability quickly decreases as the number of components increases.

Reliability with Parallel Systems

The problem with parallel systems is that the 'law of diminishing returns' operates. The rate of increase in reliability with each additional component decreases as the number of components increases.

Most systems are combinations of series and parallel systems